Q: Why do the Sun and planets have magnetic fields?

Dana Longcope
Montana State University

w/ liberal “borrowing” from Bagenal, Stanley, Christensen, Schrijver, Charbonneau, …

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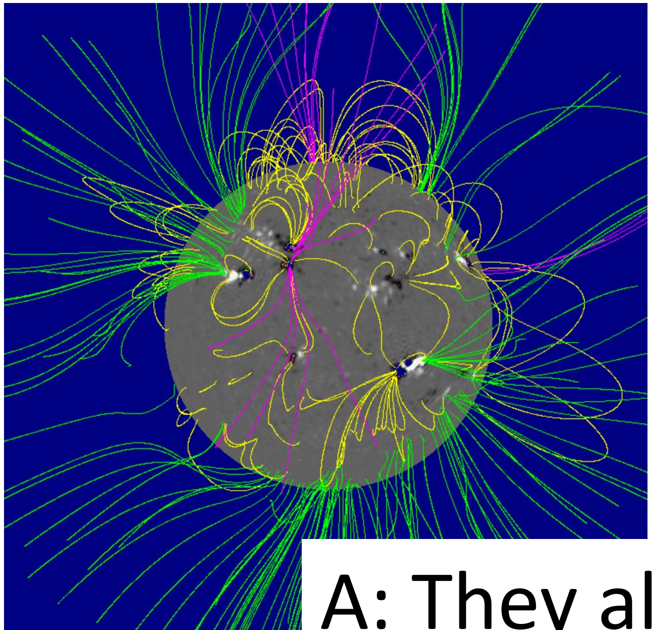
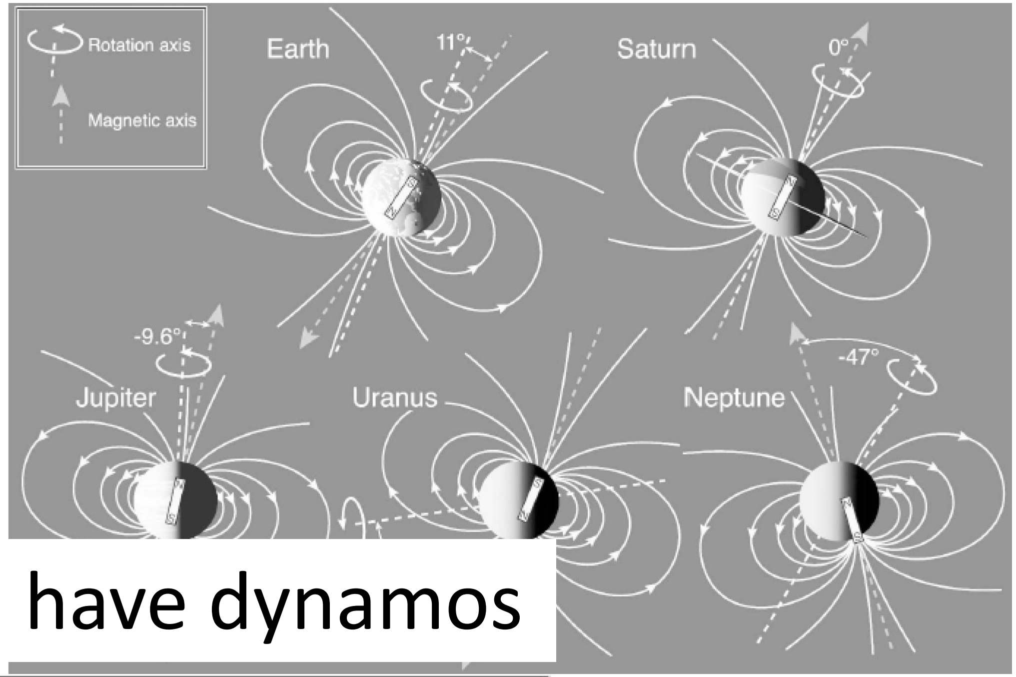
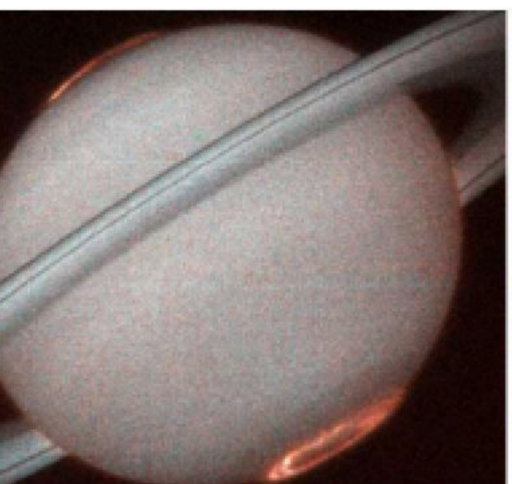
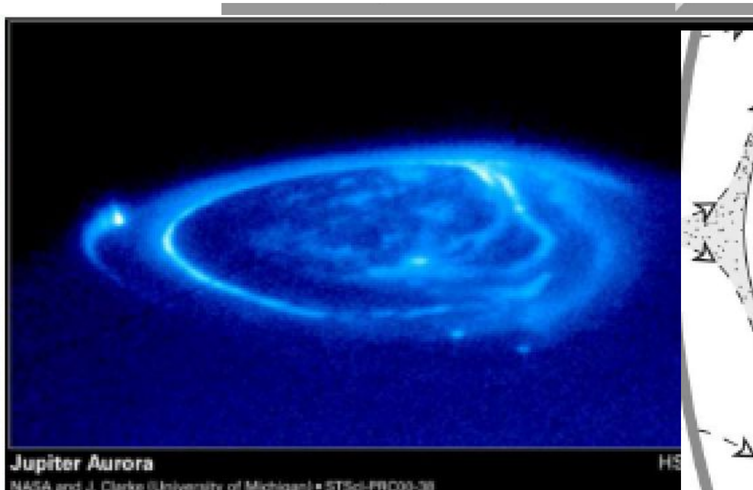
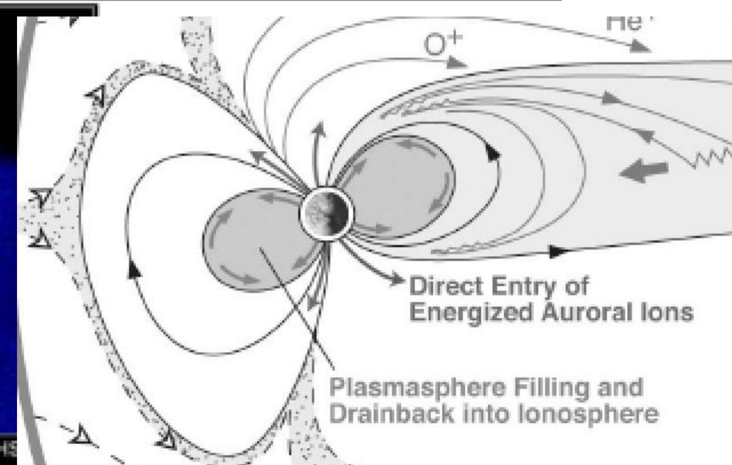




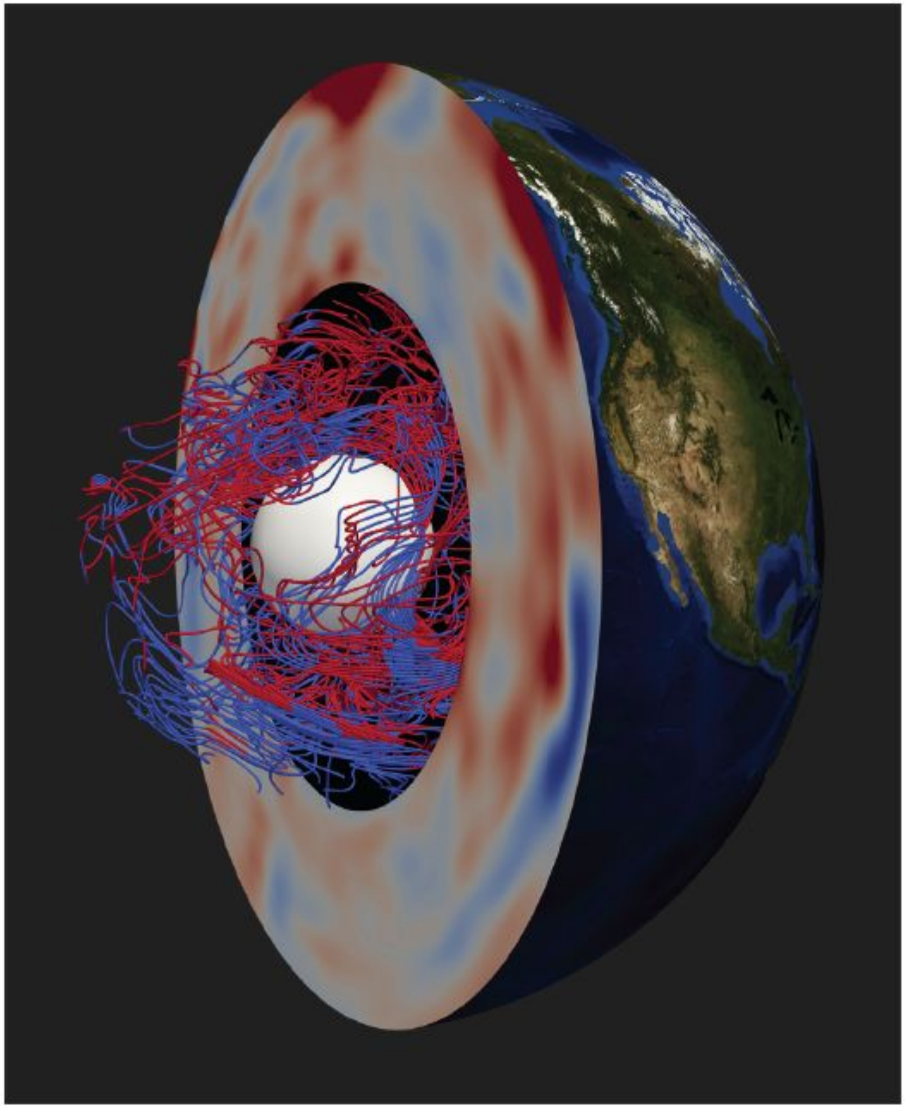
A: They all have dynamos







Dynamo Ingredients



(1)electrically conducting fluid

- Plasma (stars)
- Liquid iron (terrestrial planets)
- Metallic hydrogen (gas giants)
- Ionized water (ice giants)

(2) fluid must have complex motions

- Complex turbulent flows
- Rotation: breaks mirror-symmetry
  not required, but needed for large-scale, organized fields

(3) motions must be vigorous enough

- Figure of merit: Magnetic Reynold's #

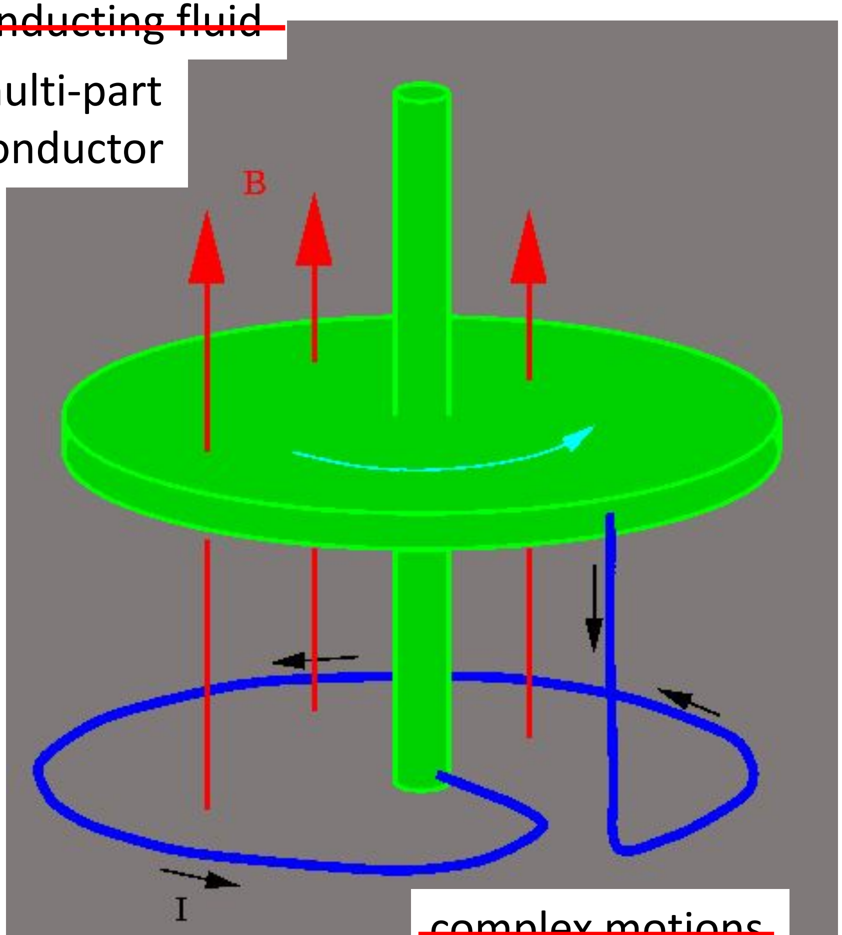
Rm = velocity × size × conductivity

From Stanley 2013

A Toy w/ all ingredients

~~conducting fluid~~

multi-part conductor



~~complex motions~~

lack of mirror symmetry

differential motion of parts

$$V_{\text{disk}} = \int_0^{\ell} vB\,dr = \int_0^{\ell} (\Omega r)\,B\,dr = \tfrac{1}{2\pi}\Omega\Phi_{\text{disk}}$$

$$IR = \underbrace{\boxed{\frac{\Omega}{2\pi}\Phi_{\text{disk}}}}_{\text{motional EMF}} - \underbrace{\boxed{L\frac{dI}{dt}}}_{\text{back EMF}} = \frac{\Omega}{2\pi}M_{w,d}I - L\frac{dI}{dt}$$

$$\frac{dI}{dt} = \left(\underbrace{\boxed{\frac{\Omega}{2\pi}\frac{M_{w,d}}{L}}}_{\text{generation}} - \underbrace{\boxed{\frac{R}{L}}}_{\text{dissipation}}\right)I = \gamma I \qquad I(t) = I_0 e^{\gamma t}$$

Growth: $\gamma > 0 \Leftrightarrow \Omega > 2\pi \dfrac{R}{M_{w,d}}$

$$\frac{v}{\ell} > 2\pi \frac{1/\sigma\ell}{\mu_0 \ell} = \frac{2\pi}{\mu_0 \sigma \ell^2}$$

Growth: $Rm = \mu_0 \sigma \ell v \; > \; 2\pi$

Reality: Conducting fluid – MHD

If **B** is weak: kinematic equations

Fluid dynamics

$$\begin{cases} \dfrac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \\ \rho \dfrac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \rho \mathbf{g} + \nabla \cdot \overleftrightarrow{\sigma} \\ \rho c_v \left(\dfrac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = -\tfrac{2}{3} T \nabla \cdot \mathbf{v} + \nabla \mathbf{v} : \overleftrightarrow{\sigma} + \dot{Q} \end{cases}$$

Traditional (neutral) fluid – solve first

Faraday's + Ohm's laws

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{B} = (\mathbf{B} \cdot \nabla) \mathbf{v} - \mathbf{B} (\nabla \cdot \mathbf{v}) + \eta \nabla^2 \mathbf{B}$$

Linear equation **for** **B**(**x**,t) – solve w/ known **v**(**x**,t)

Dynamo action in MHD

$$\overbrace{\frac{\partial \mathbf{B}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{B}}^{\frac{D\mathbf{B}}{Dt}}=\overbrace{(\mathbf{B}\cdot\nabla)\mathbf{v}-\mathbf{B}(\nabla\cdot\mathbf{v})}^{\mathbf{B}\cdot[\nabla\mathbf{v}-\mathbf{I}(\nabla\cdot\mathbf{v})]=\mathbf{B}\cdot\mathbf{M}}+\eta\nabla^2\mathbf{B}$$

If **M** has a positive eigenvalue $\lambda > 0$

B can grow exponentially: **DYNAMO ACTION**

- **B** → -**B** : same e-vector → same λ
- Reverse velocity AND reflect in mirror → λ → λ
- Do one and not the other → λ → -λ

$$\gamma \sim \overset{\lambda}{\frac{v}{\ell}}-\overset{\eta\nabla^2}{\frac{\eta}{\ell^2}}=\frac{v}{\ell}\left(1-\frac{\eta}{\ell v}\right)$$

Growth: $Rm=\dfrac{\ell v}{\eta}=\mu_0 \ell v\sigma > 1$

Q: What kind of flow has λ > 0?

- Turbulent flows have pos. Lyapunov exponent: $\lambda > 0$
  - tend to stretch balls into strands 
  - tend to amplify fields
- Conditions for turbulence:
  - driving: e.g. Rayleigh-Taylor instability
  - viscosity fights driving – must be small

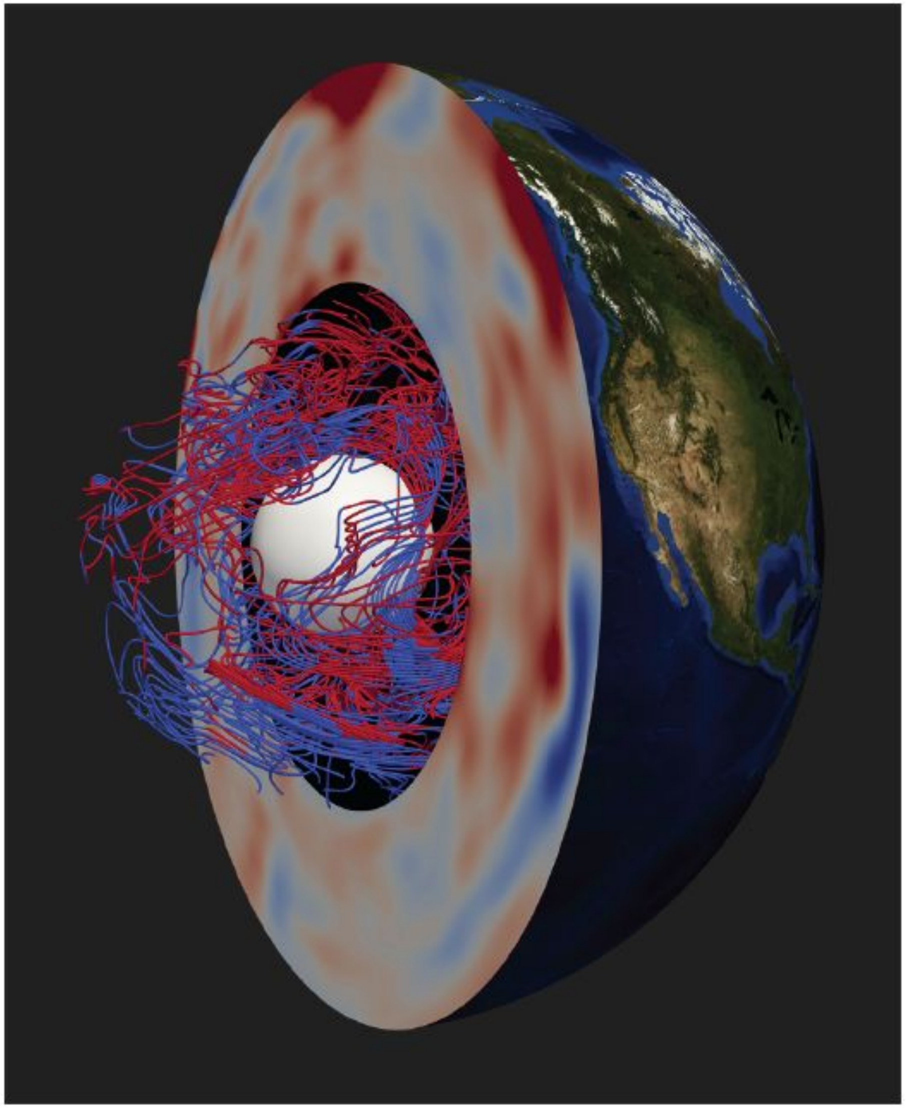
$$\mathrm{Re} = \frac{L v}{\upsilon} >> 1$$

- Rotation can organize turbulence:
  align stretching direction → azimuthal
  (toroidal) – known as Ω-effect
  – must be significant w.r.t. fluid motion

$$\mathrm{R}o = \frac{v}{L\Omega} << 1$$

	η [m^2/s]	υ [m^2/s]	L [m]	v [m/s]	Ω [rad/s]	Rm	Re	Ro
Sun (CZ)	1	10^{-2}	10^{8}	1	10^{-6}	10^{8}	10^{10}	10^{-2}
Earth (core)	1	10^{-5}	10^{6}	10^{-4}	10^{-4}	10^{2}	10^{7}	10^{-6}

Dynamo Ingredients



From Stanley 2013

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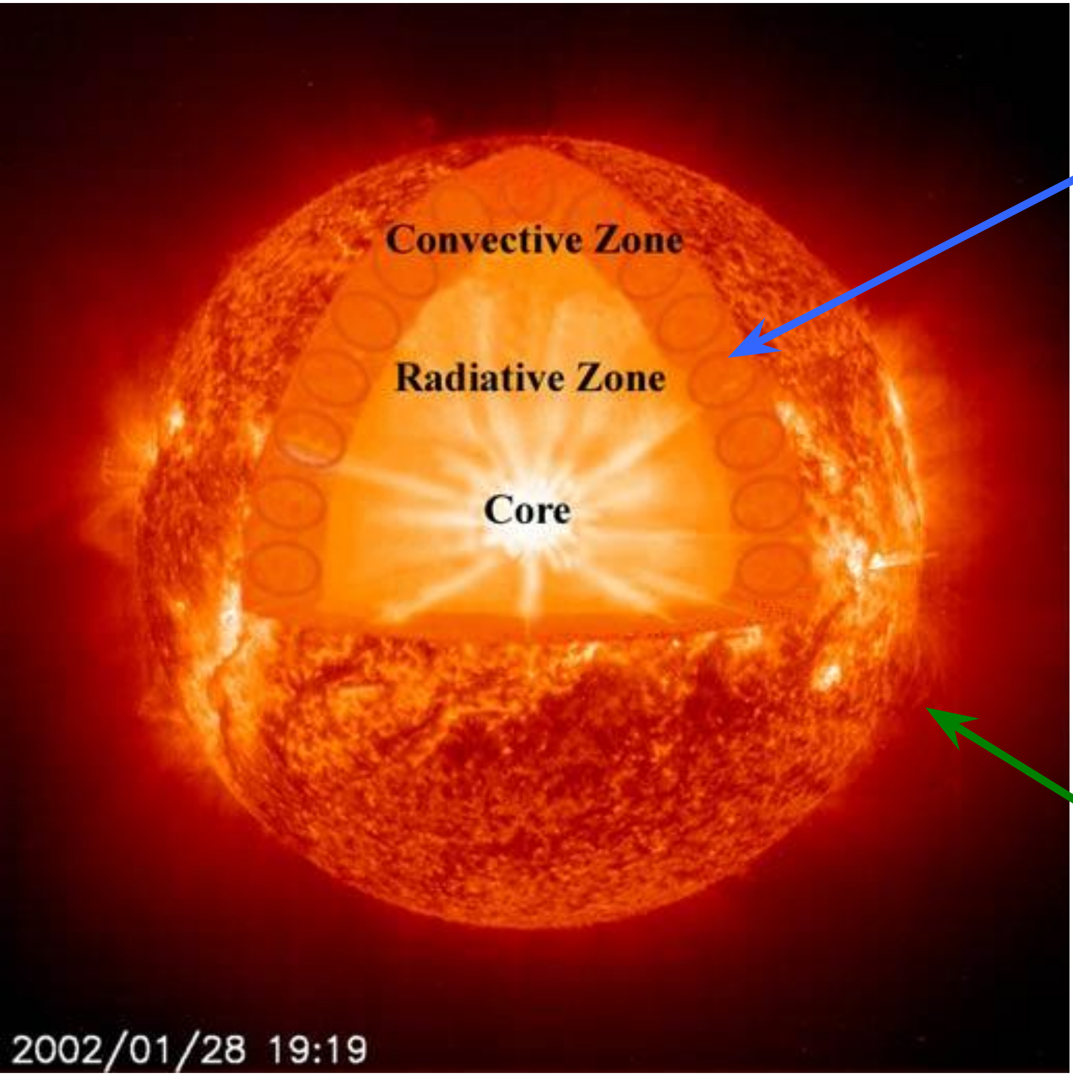
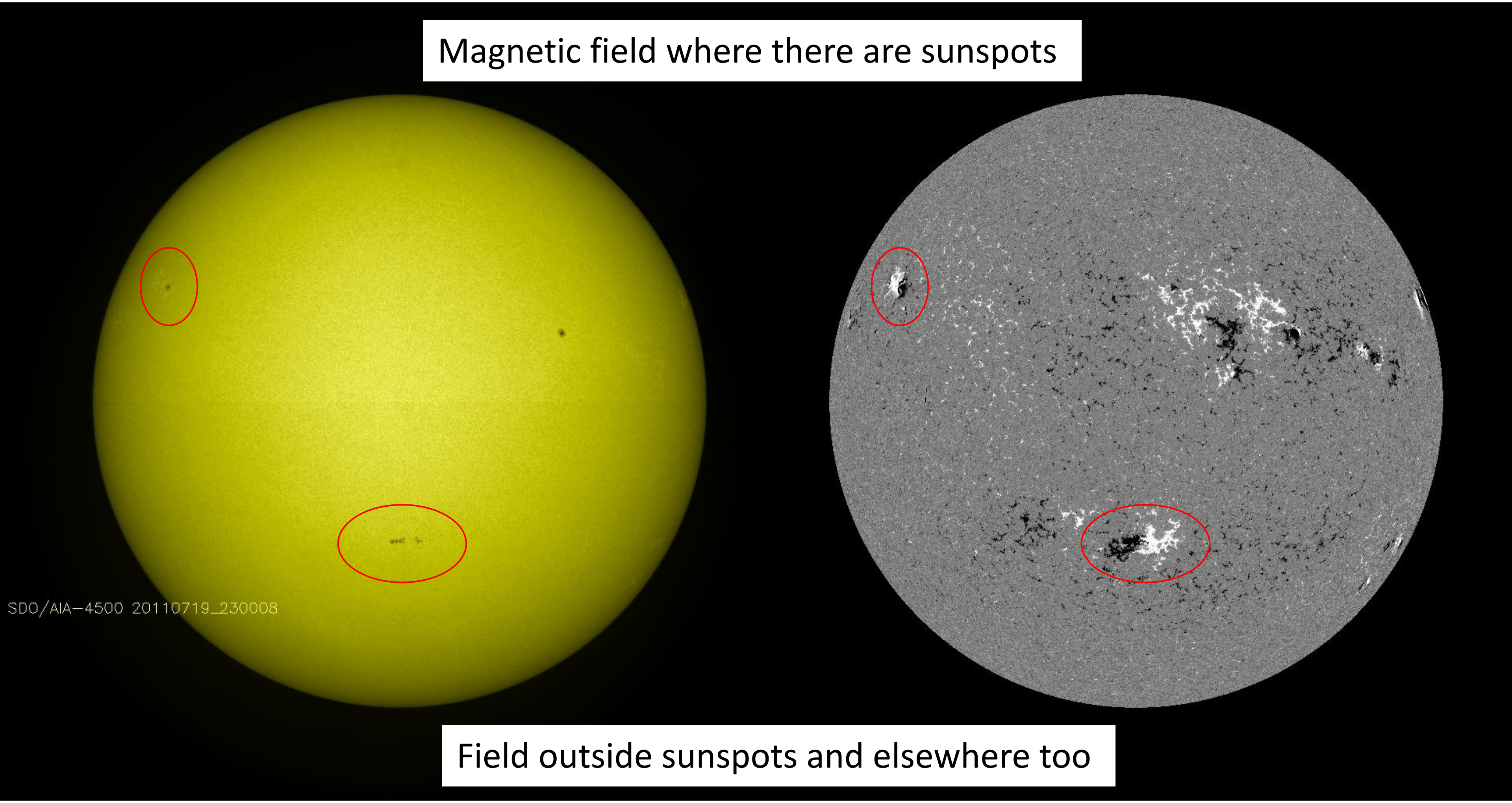
Rm = velocity × size × conductivity

How it works in the Sun



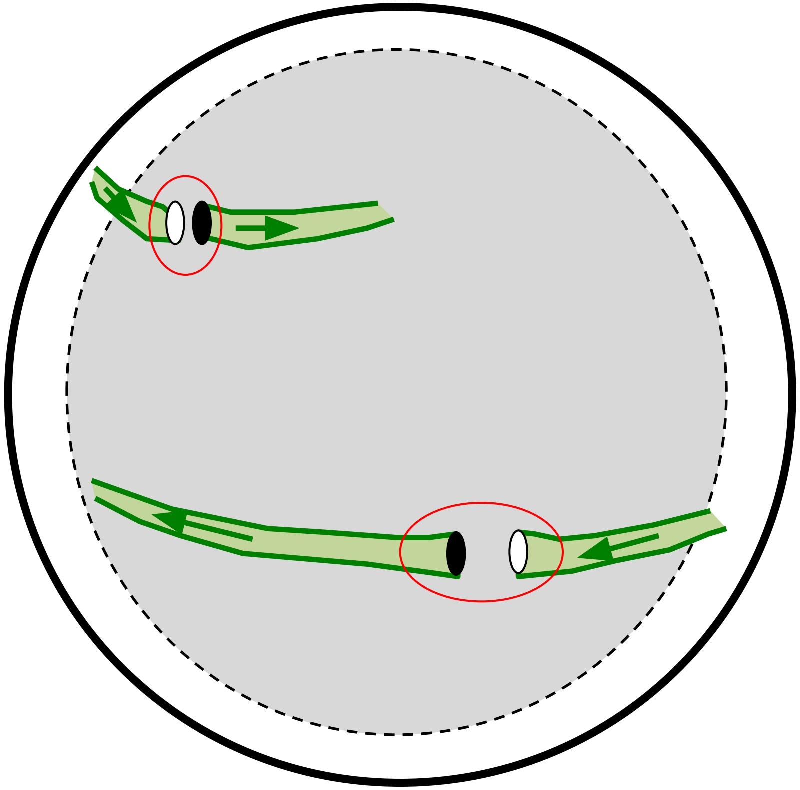
- Entire Star: H/He plasma
- Convection Zone (CZ)
  - Outer 200,000 km
  - Turbulence:
    $Re = 10^{10}$
  - Thermally driven
  - Good conductor
    $Rm = 10^{8}$
  - Rotation effective
    $Ro = 10^{-2}$
- Corona – conductive but tenuous:
  J smaller (~0?)

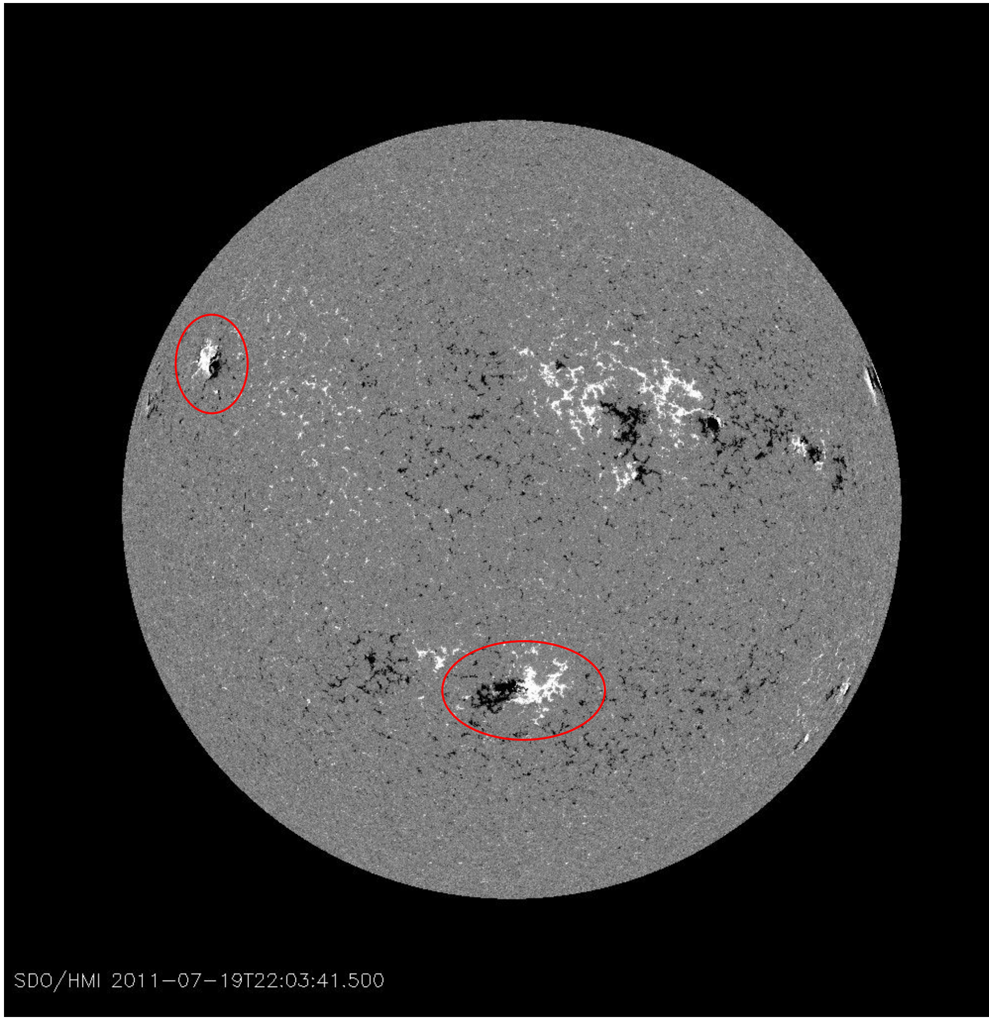
Evidence of the dynamo



Evidence of the dynamo

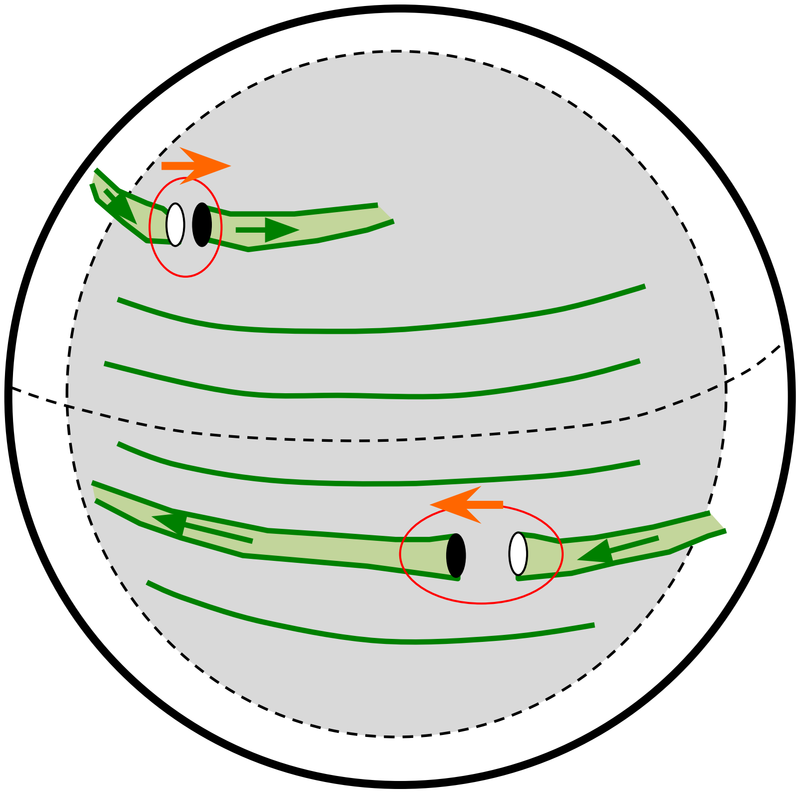
Field is **fibril**

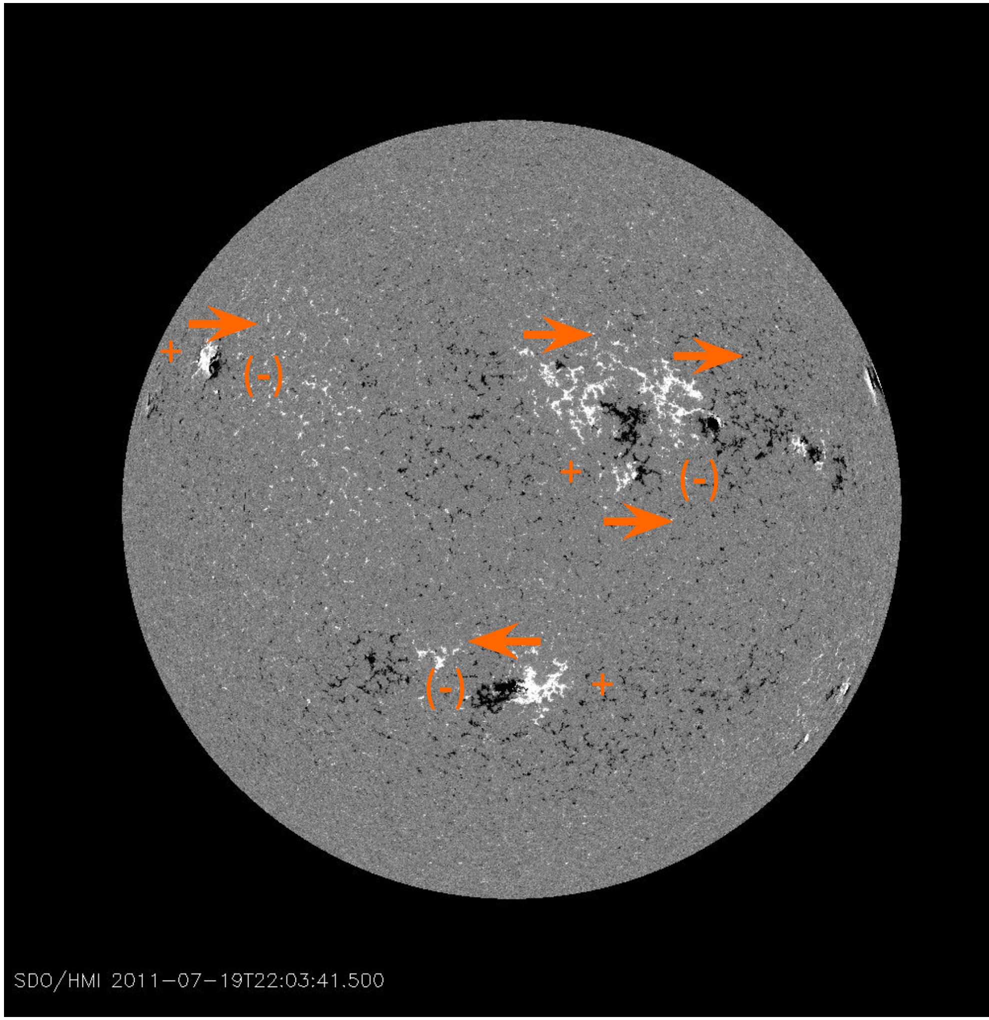




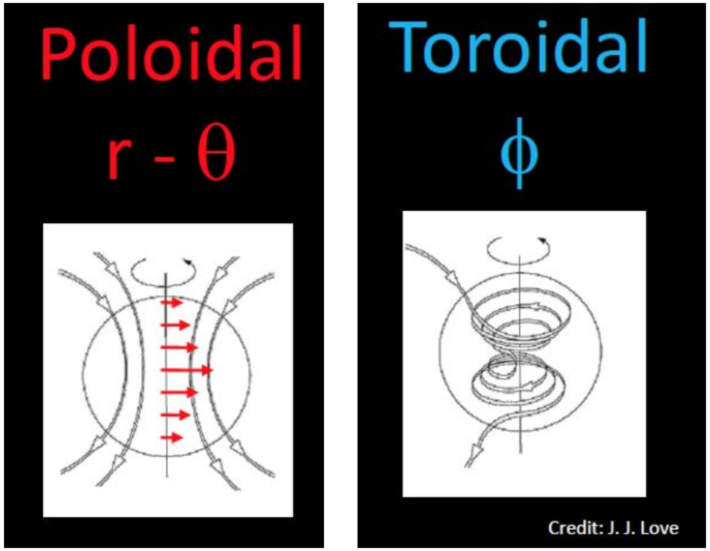
Evidence of the dynamo

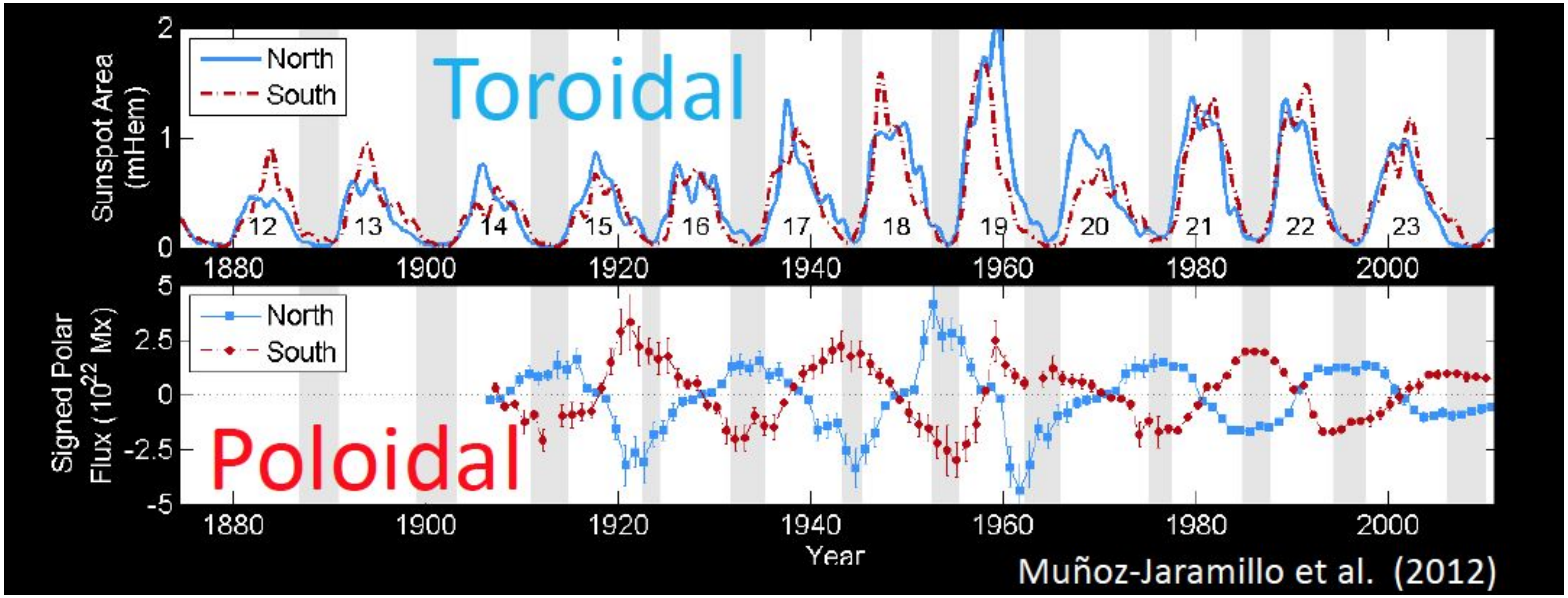
Field orientation: mostly **toroidal (E-W)**





An oscillator at work:

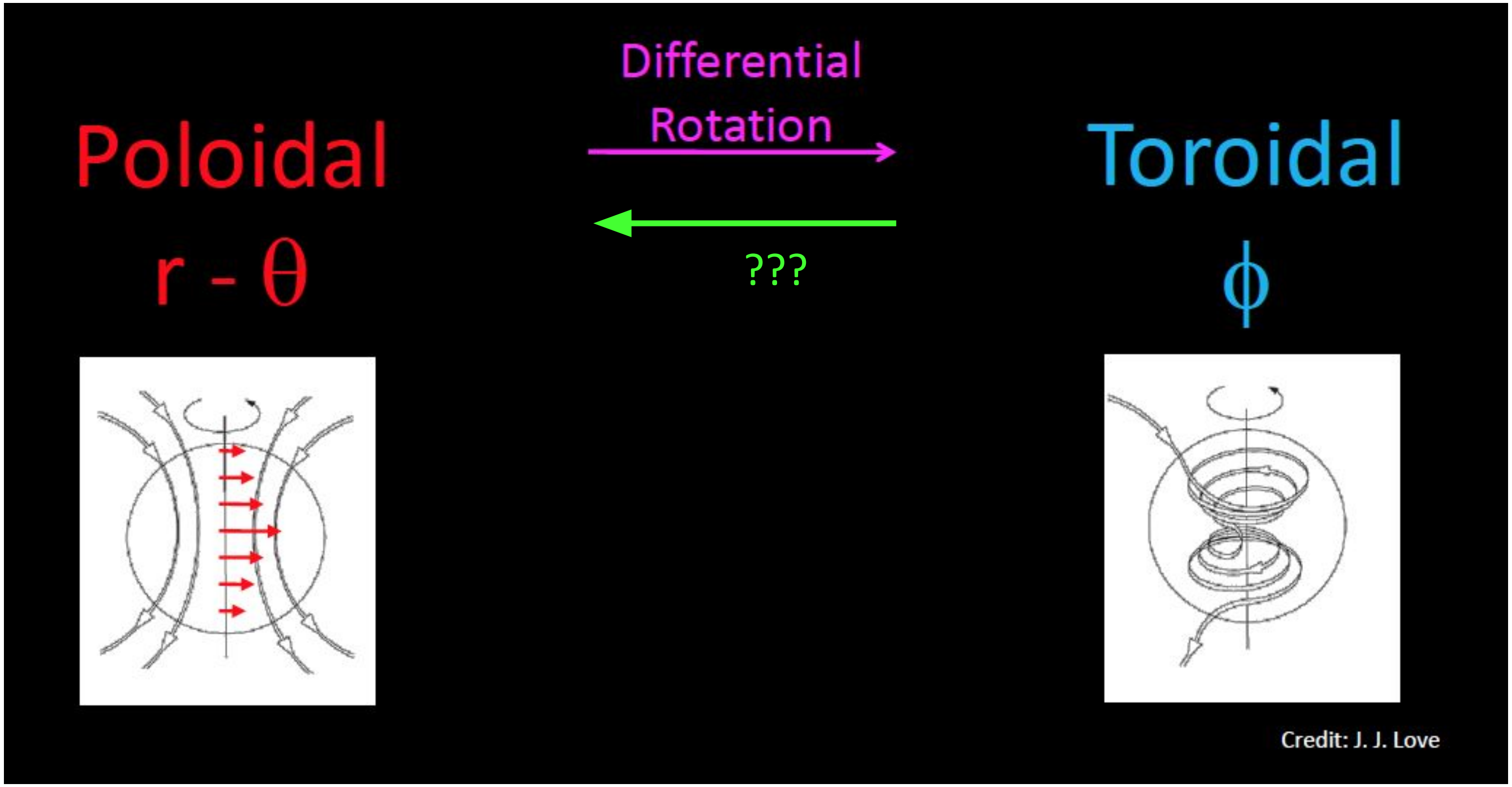




Munoz-Jaramillo lecture (2015)

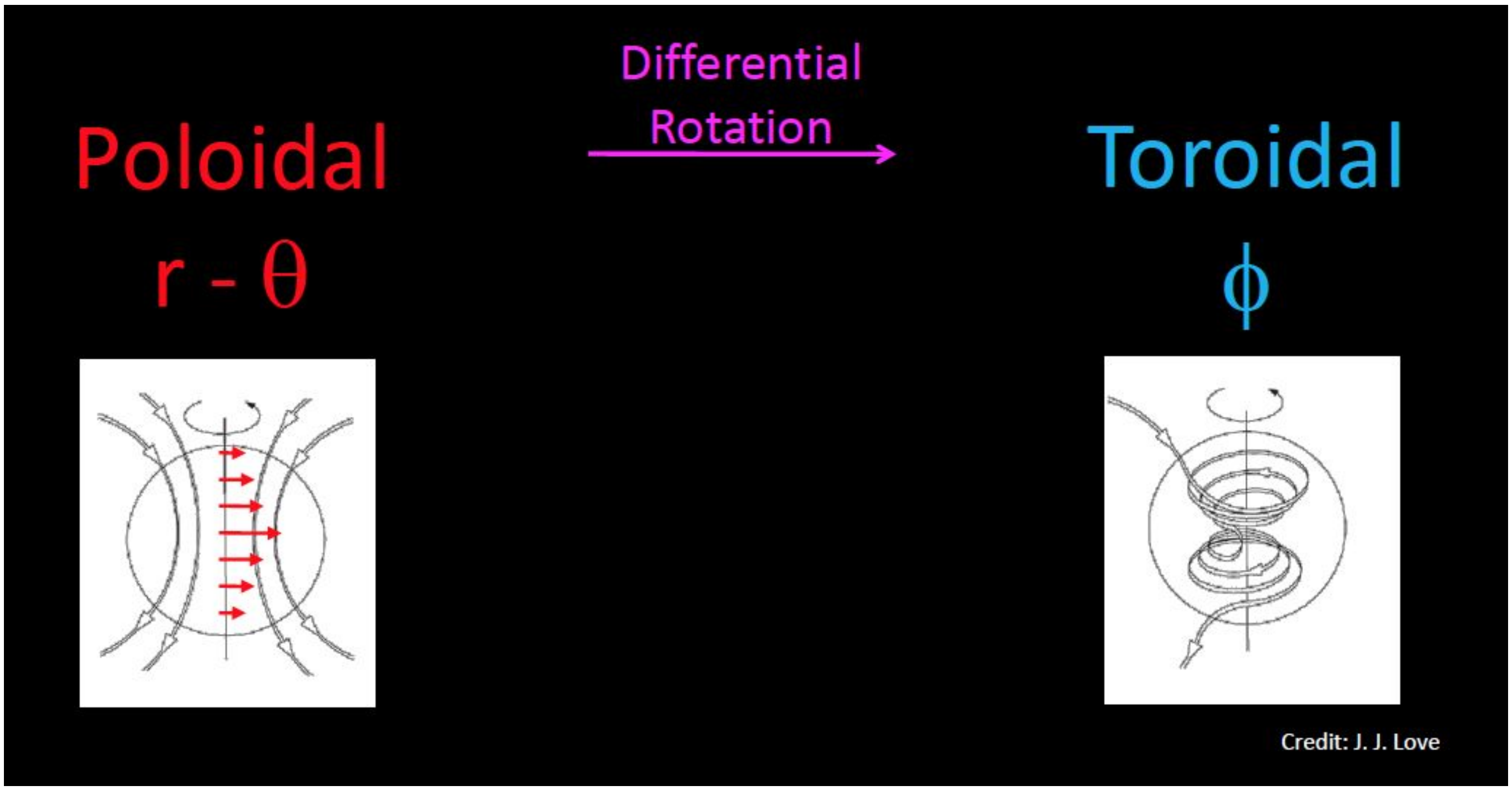
NB: B_{pol} & B_{tor} out of phase like Q & I in LC circuit (oscillator)

Solar dynamo as an oscillator



Munoz-Jaramillo lecture (2015)

Half of the oscillation…



Munoz-Jaramillo lecture (2015)

Activity 44: Estimate the time it takes for the solar equator to execute one more full rotation than the poles in the same time.

$P = 25$ d @ equator: $f = 1/25 = 0.040\ d^{-1}$

$P = 35$ d @ poles: $f = 1/35 = 0.029\ d^{-1}$

$\Delta f = 0.011\ d^{-1}$ □ $1/\Delta f = 87$ d

Check:

$N_{eq} = 87/25 = 3.5$ rotations of equator

$N_{pole} = 87/35 = 2.5$ rotations of equator

Activity 47: If we take the Sun's polar field – averaging at cycle minimum at about 5 Gauss – how long would it take to wind that field into a strength of some 10^5 G? Hint: remember the field line stretch-and-fold from Fig. 4.6, look at the illustration in Fig. 4.9, and consider 'compound interest'. [i.e. exponentials]

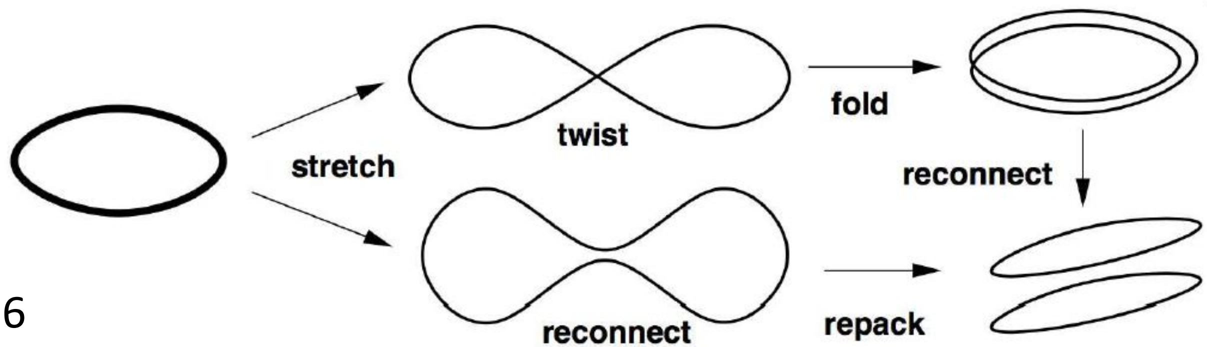


Fig. 4.6

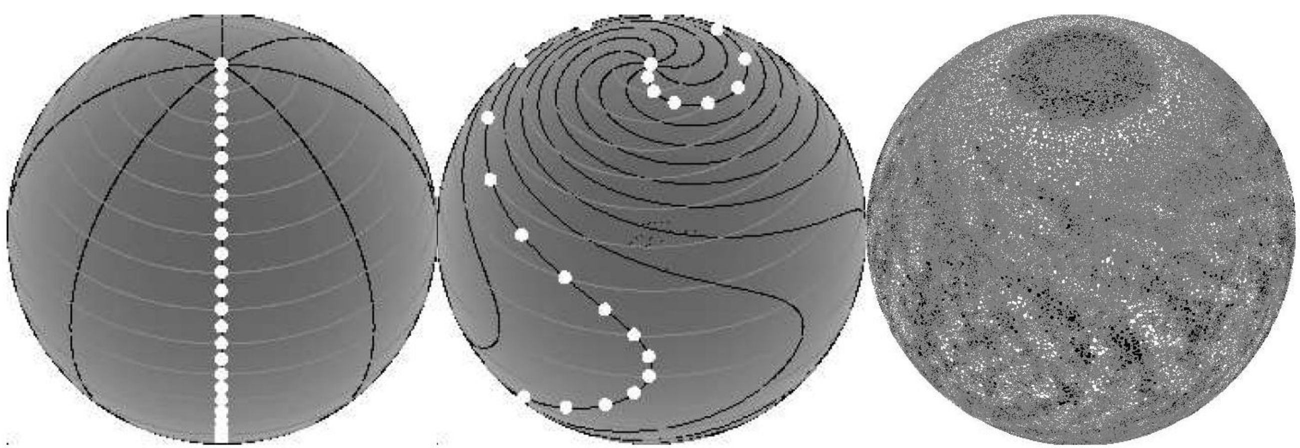


Fig. 4.9

Activity 47: p. 86: If we take the Sun's polar field – averaging at cycle minimum at about 5 Gauss – how long would it take to wind that field into a strength of some 10^5 G? Hint: remember the field line stretch-and-fold from Fig. 4.6, look at the illustration in Fig. 4.9, and consider 'compound interest'. [i.e. exponententials]

1 stretch+fold requires **87 days**

(one rotation of pole w.r.t. equator)

`compound interest': Field strength **doubles** every 87 days

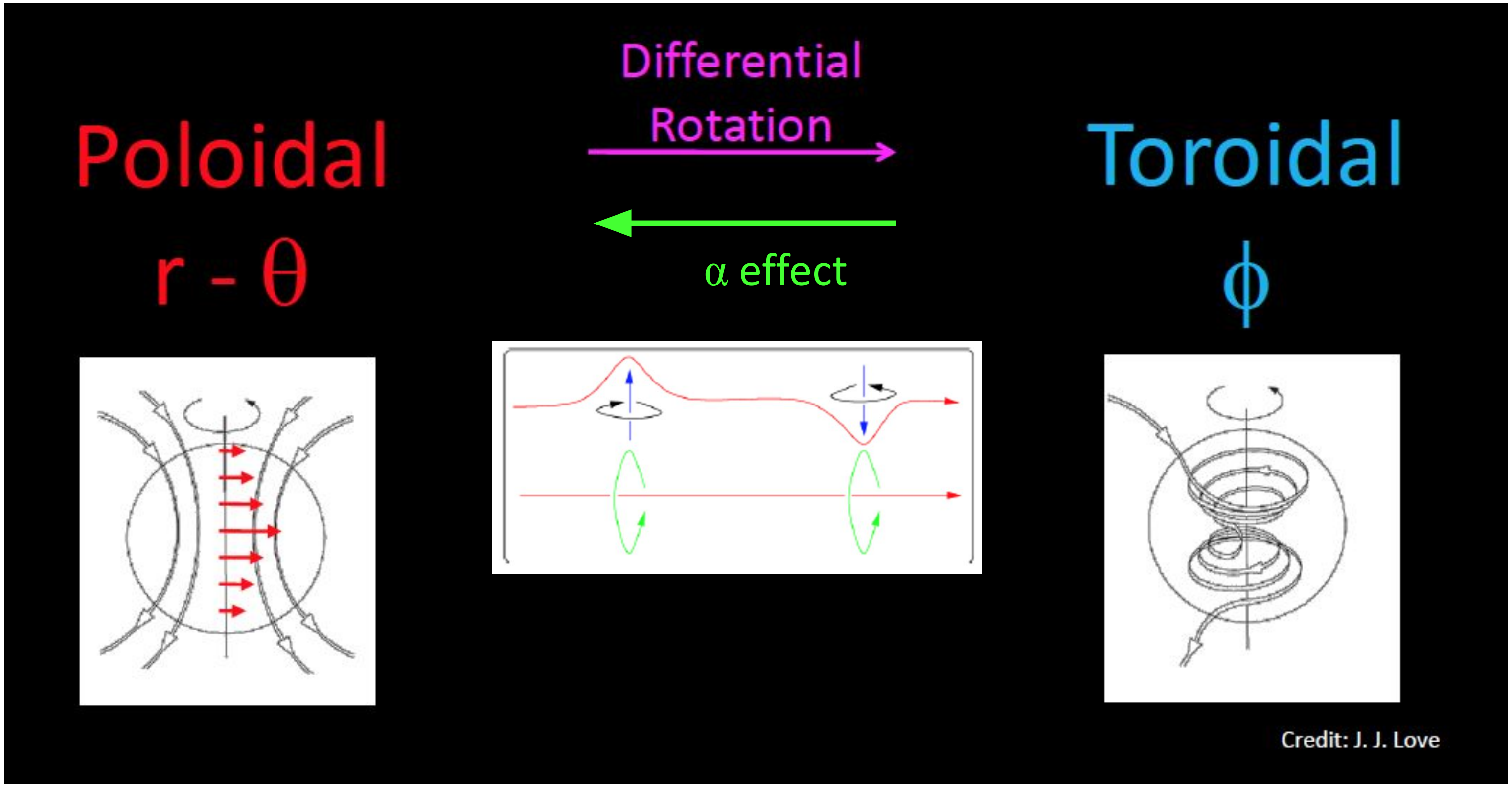
 e-folding time = 87/ln(2) = 125 days

$B = 5\text{ G} \times \exp(t/125) = 10^5\text{ G}$

$t = 125\text{ d} \times \ln(10^5/5) = 1{,}240$ days = 3.4 years

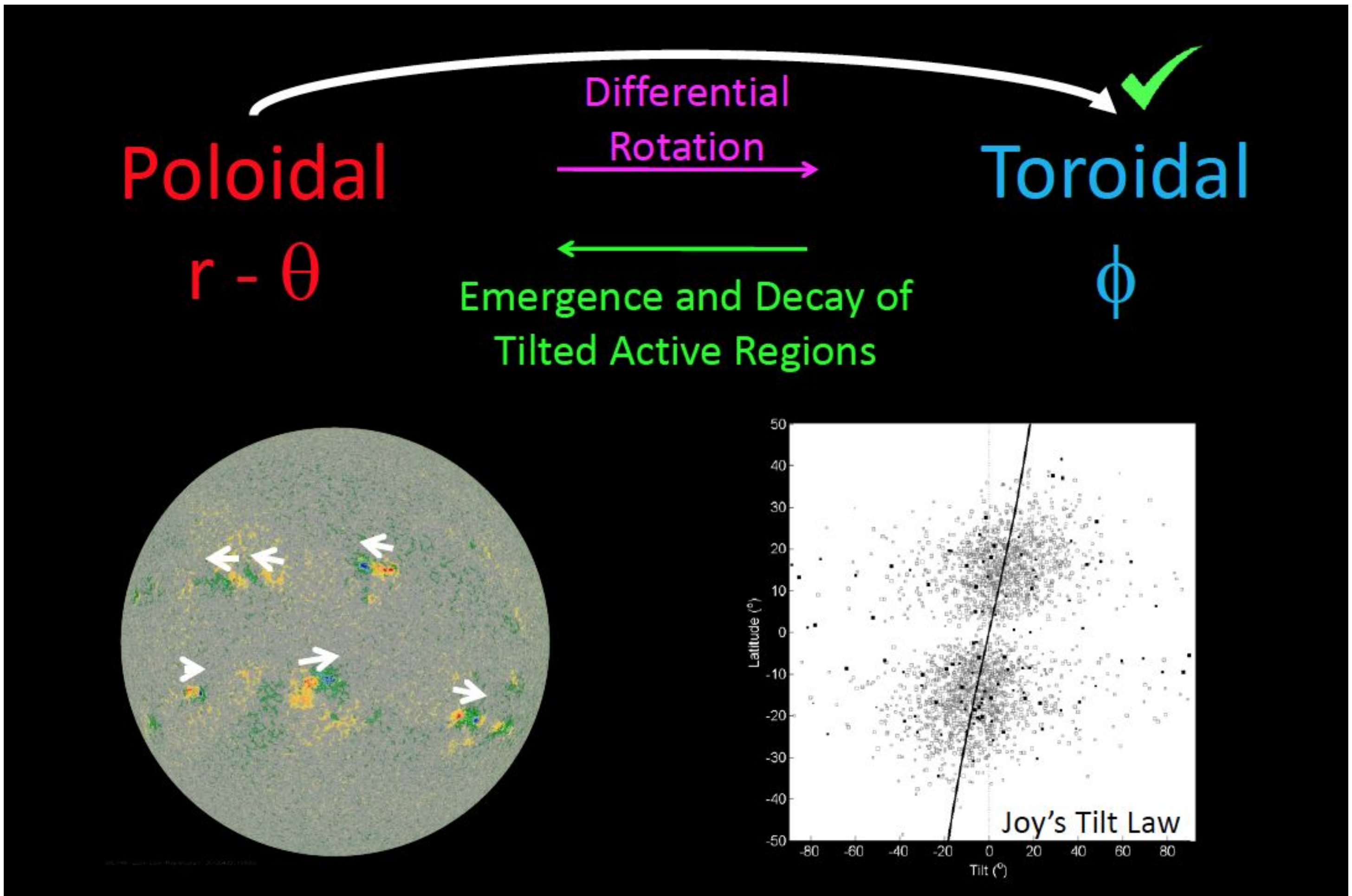
The other half … mean field theory

(see lecture by Prof. Bhattacharjee)

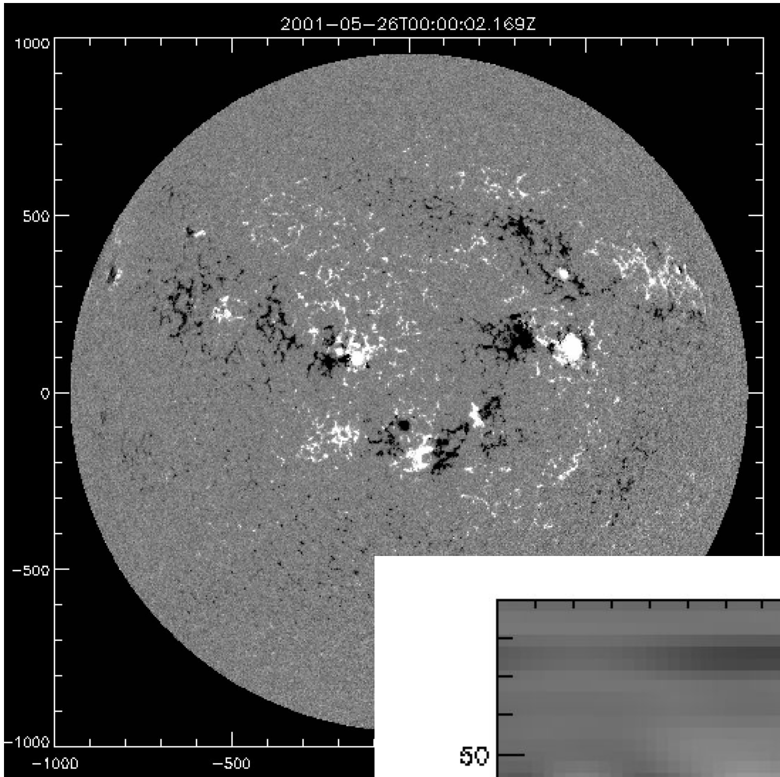


Munoz-Jaramillo lecture (2015)

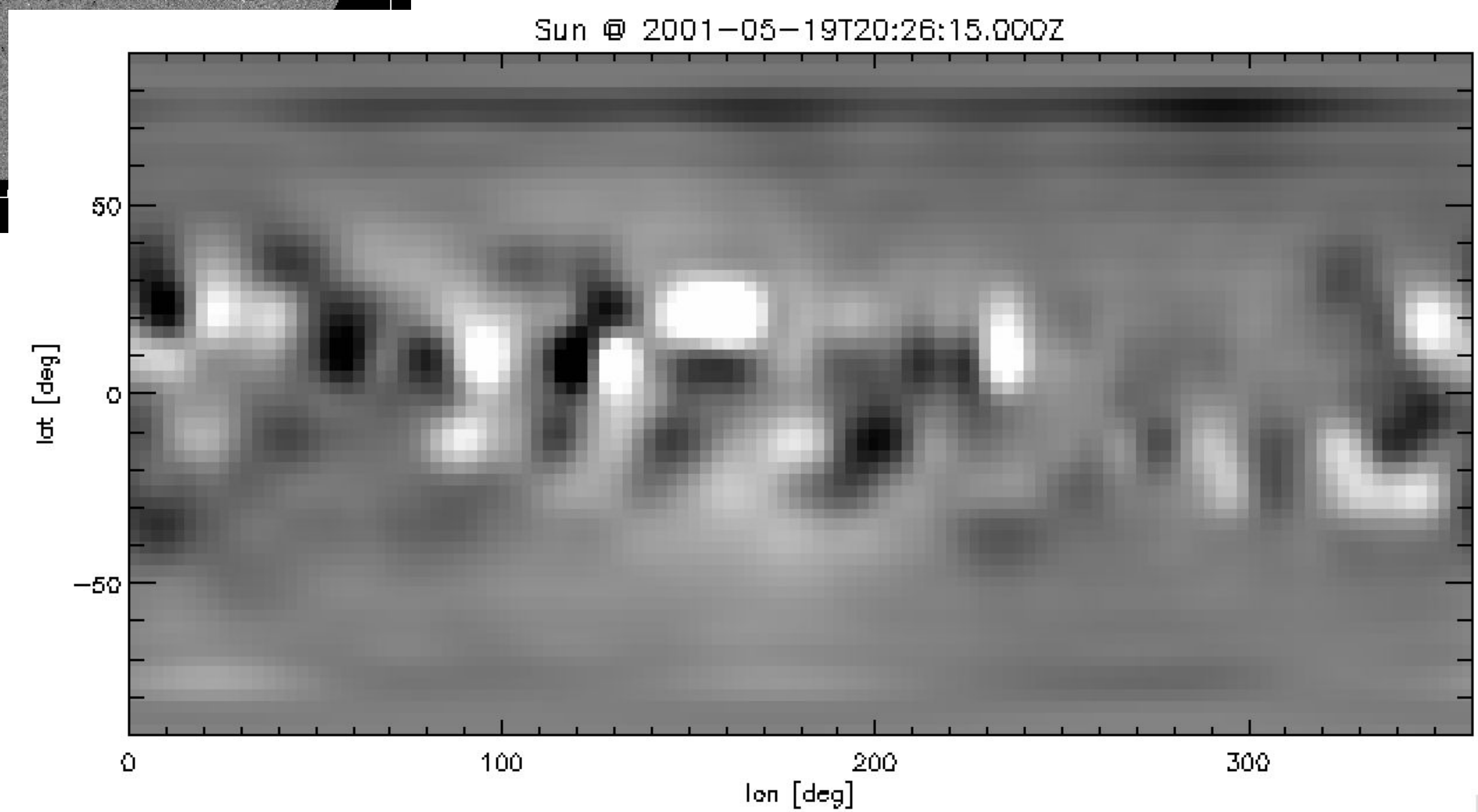
The other half…
Babcock-Leighton model



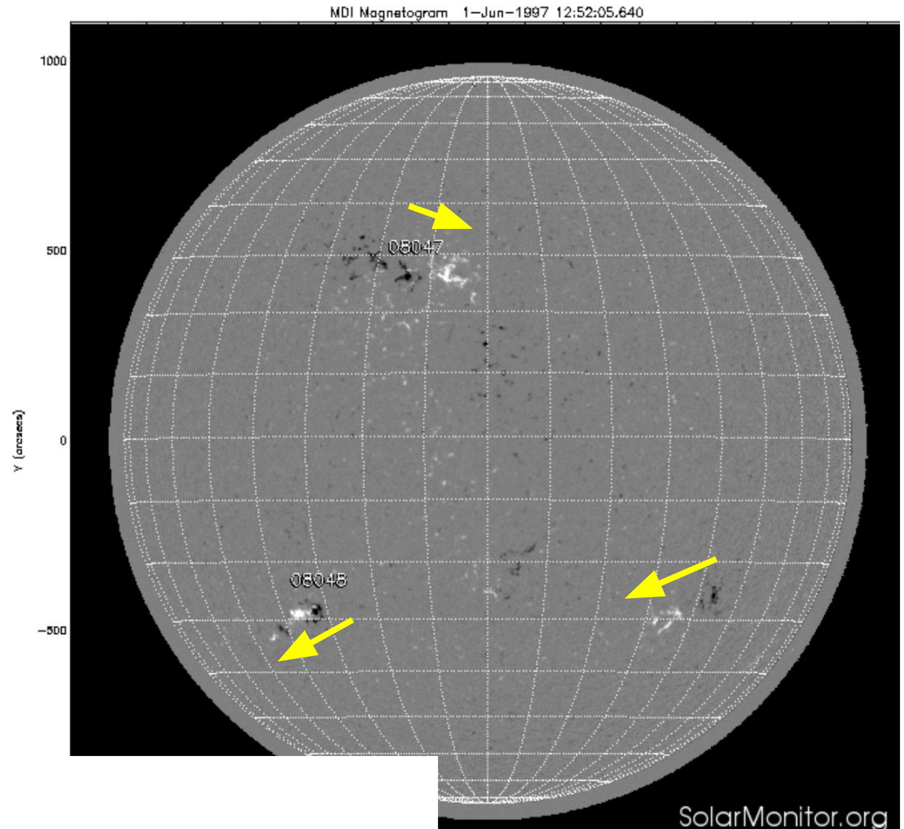
Munoz-Jaramillo lecture (2015)



Synoptic plot: unwrapped view built up over time



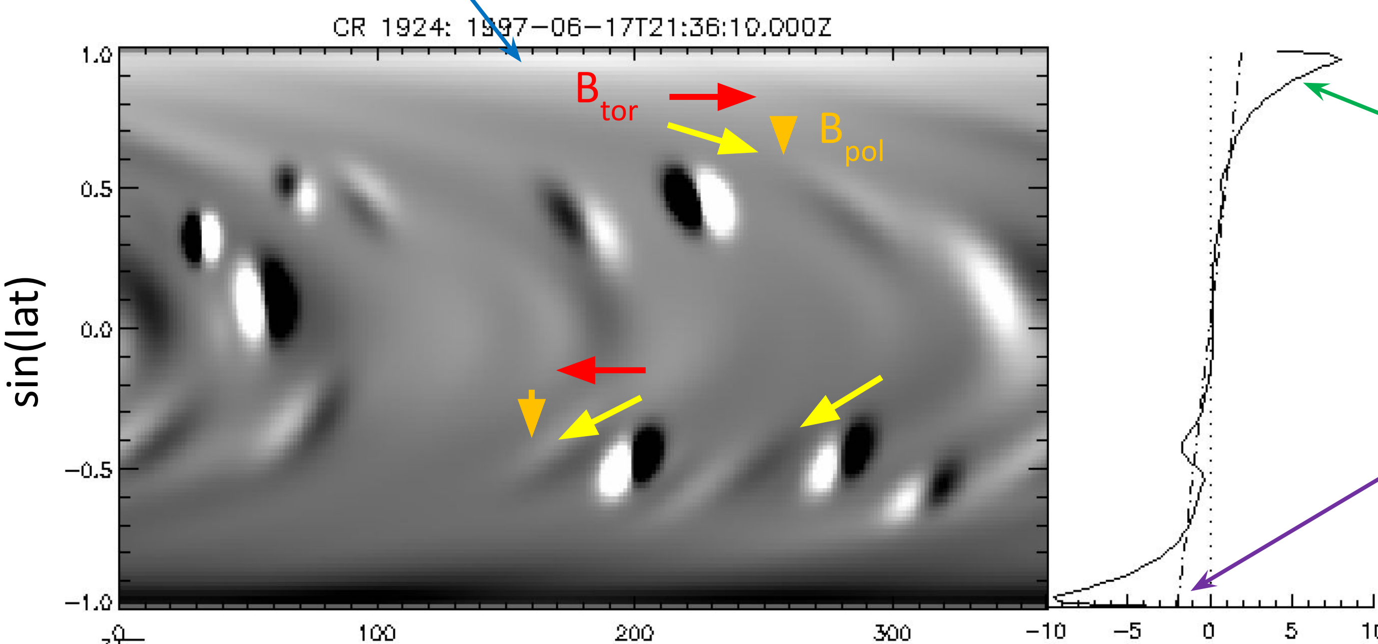
Tilted active regions contribute to magnetic dipole moment – in sense to reverse it



Hale's Law: toroidal field from diff. rotation

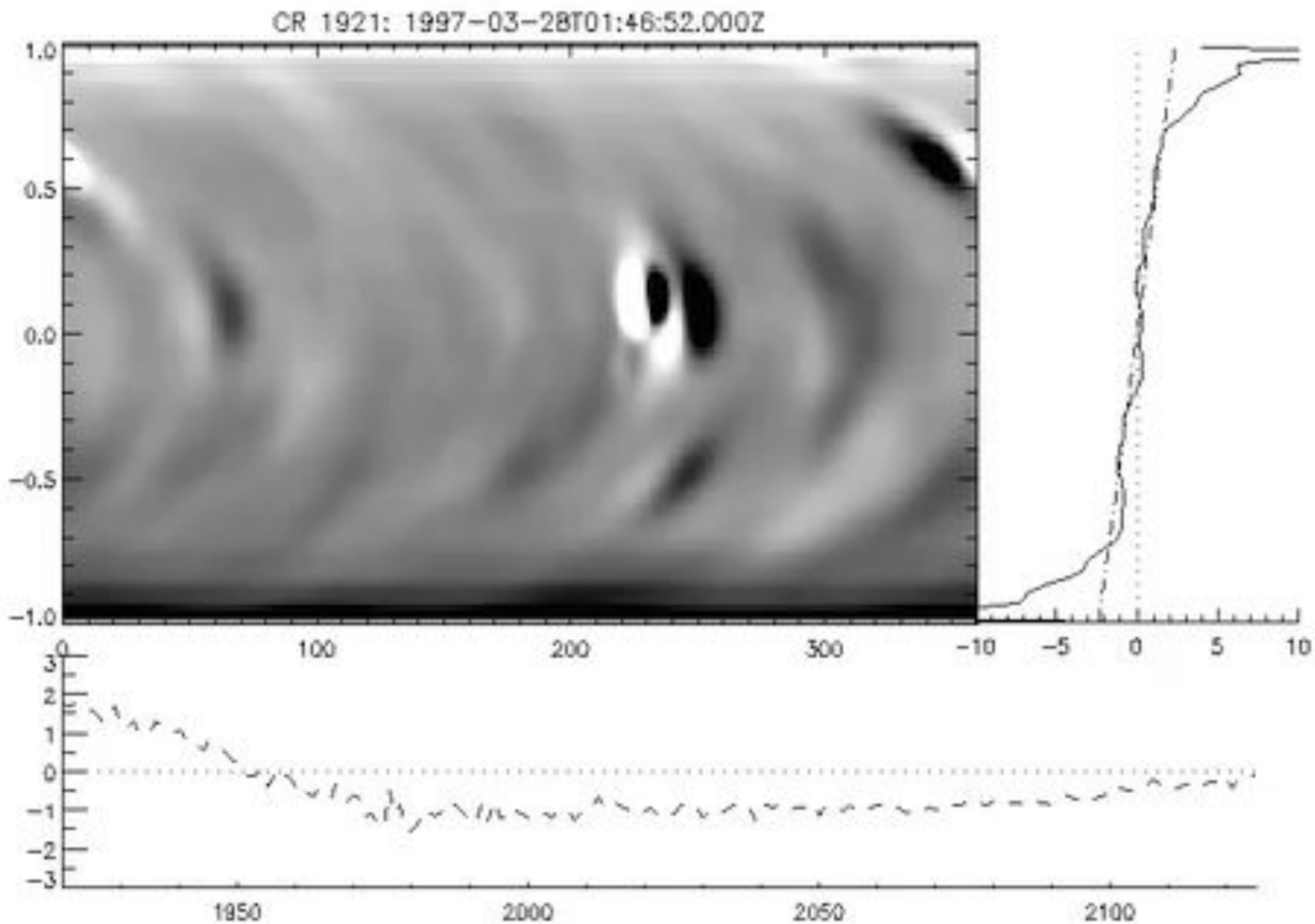
Joy's Law: tilt of bipole
□ poloidal field

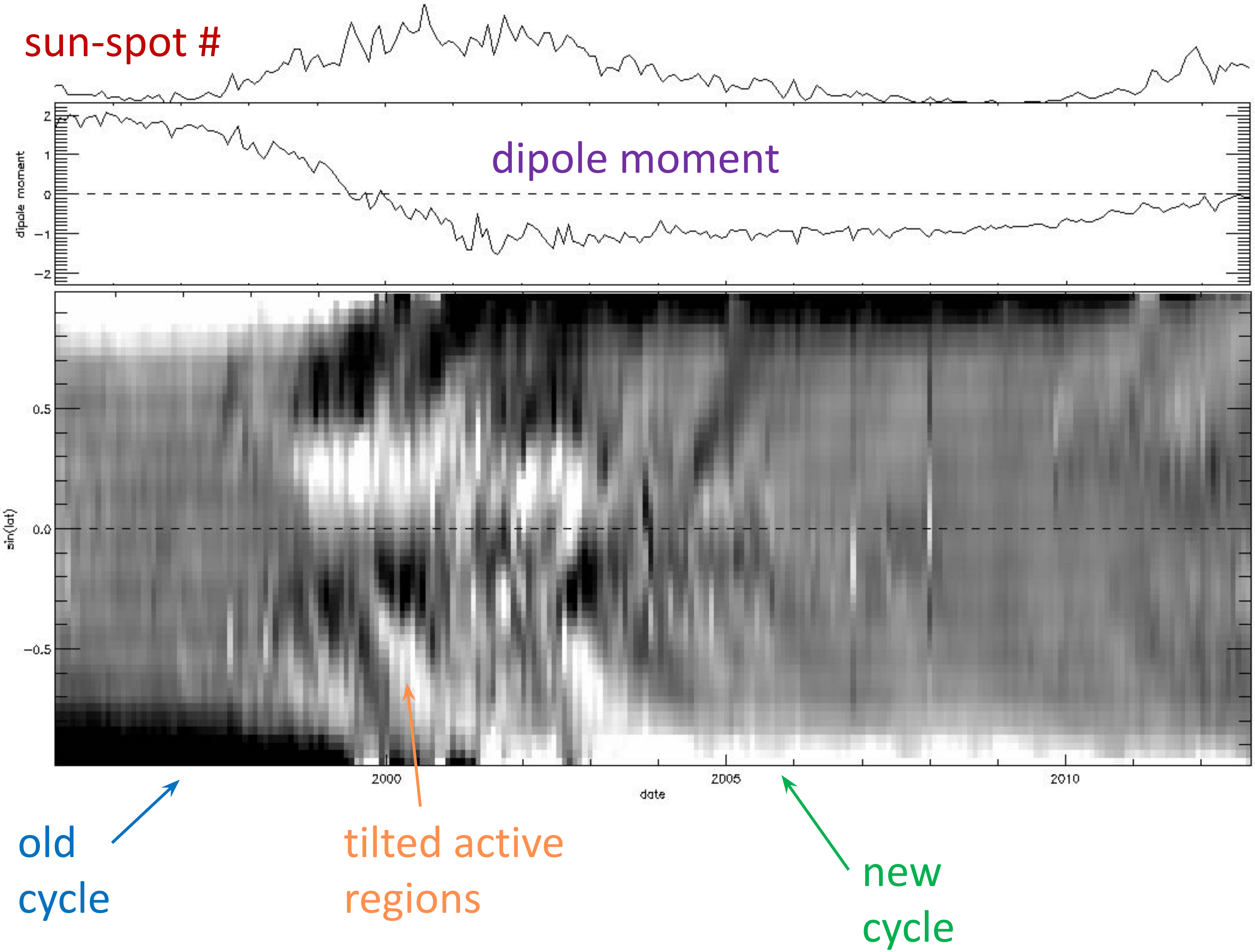
remnant of last cycle



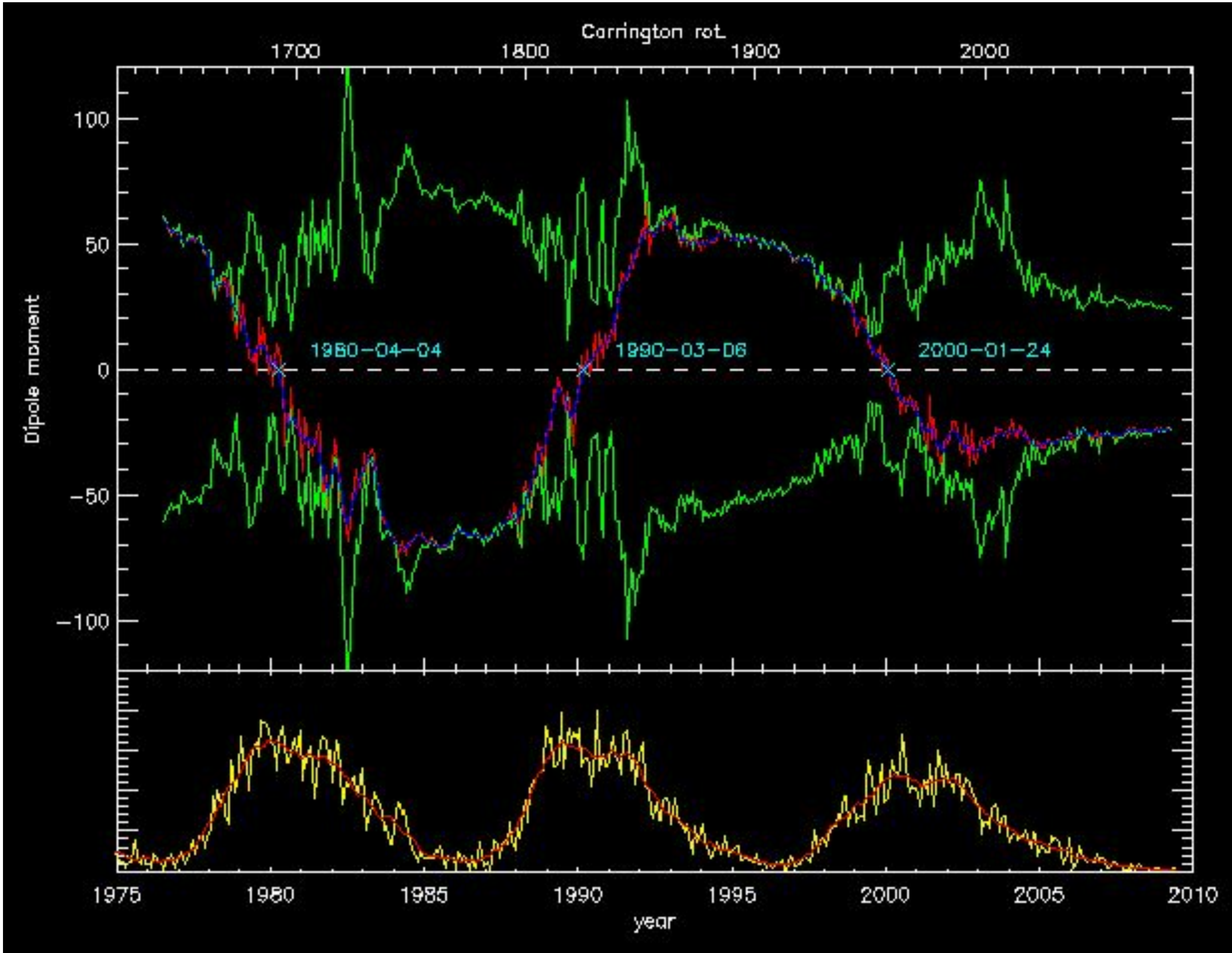
avg-ed over φ

dipole component


CR 1921: 1997-03-28T01:46:52.000Z
1.0
0.5
0.0
−0.5
−1.0
0
100
200
300
−10
−5
0
5
10
3
2
1
0
−1
−2
−3
1950
2000
2050
2100



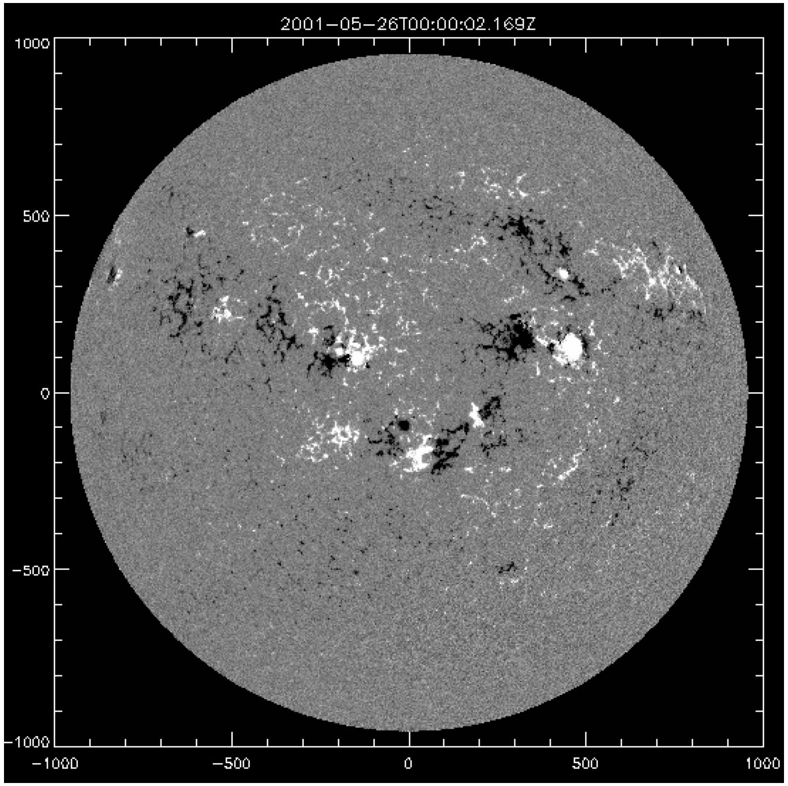
sun-spot #
dipole moment
2
1
0
−1
−2
0.5
0.0
−0.5
sin(lat)
2000
2005
2010
date
old
cycle
tilted active
regions
new
cycle



Carrington rot.
1700
1800
1900
2000
100
50
0
-50
-100
Dipole moment
1980-04-04
1990-03-06
2000-01-24
1975
1980
1985
1990
1995
2000
2005
2010
year

Activity 51: Question: with this value of D [=250 km^2/s], what is the characteristic time scale for flux to disperse over the solar surface (hint: Eq. 3.20)? [H]ow important is the meridional advection from equator to pole (with a characteristic velocity of 10 m/s) in transporting the field within the duration of a solar cycle?

$$\tau_{\mathrm{d}} \sim \frac{L_{\mathrm{t}}^2}{\eta} . \qquad (3.20)$$



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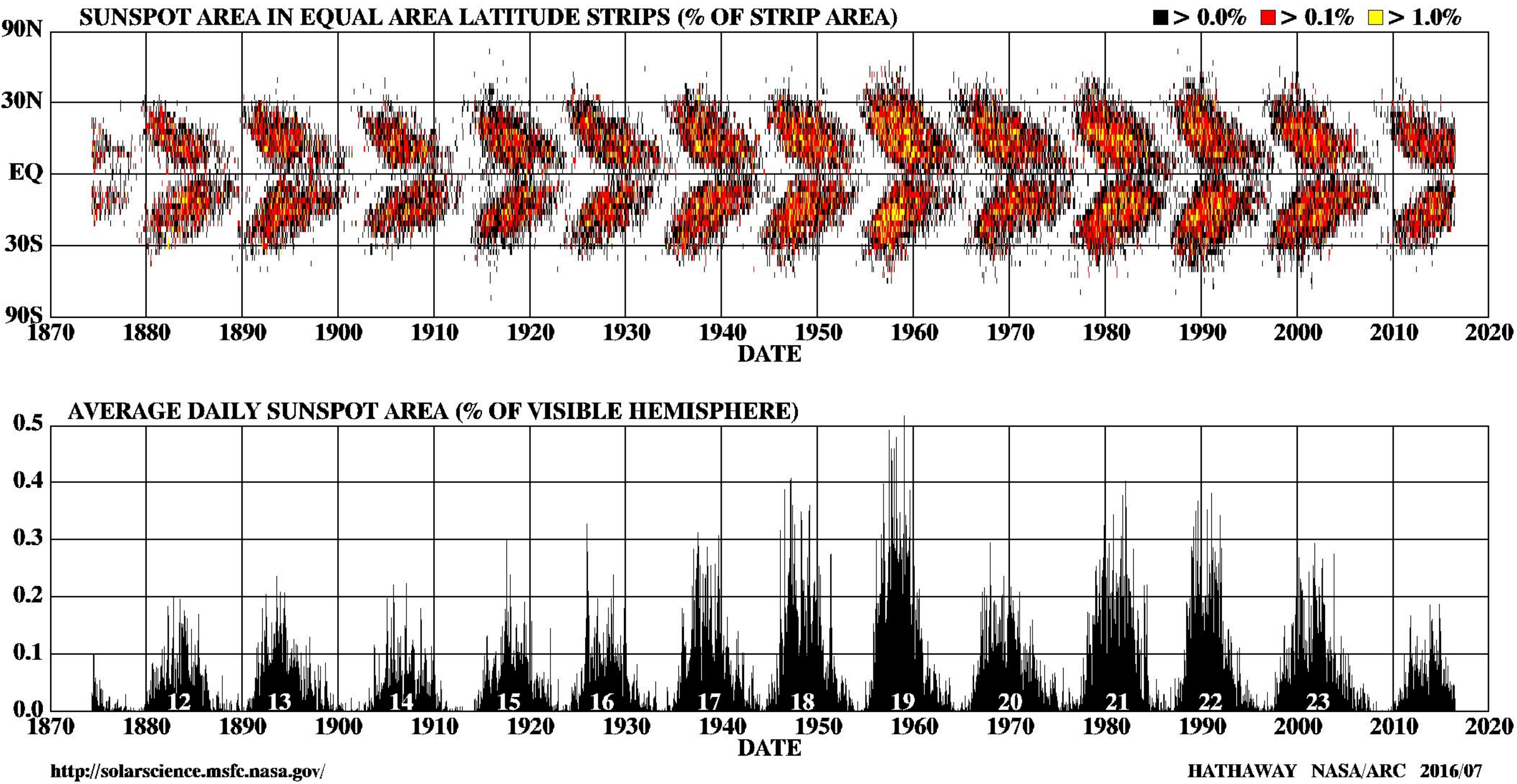
$$\tau_{\mathrm{d}} \sim \frac{L_{\mathrm{t}}^2}{\eta}. \qquad (3.20)$$

$$L_t \sim R_\odot = 7 \times 10^5 \text{ km}$$

$$\tau_d \sim L_t^2/D = 2 \times 10^9 \text{ s} = 65 \text{ y}$$

$$\textit{vs.} \quad R_\odot/v = (7 \times 10^5 \text{ km})/(10^{-2} \text{ km/s})$$
$$= 7 \times 10^7 \text{ s} = 2.3 \text{ y}$$

DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS



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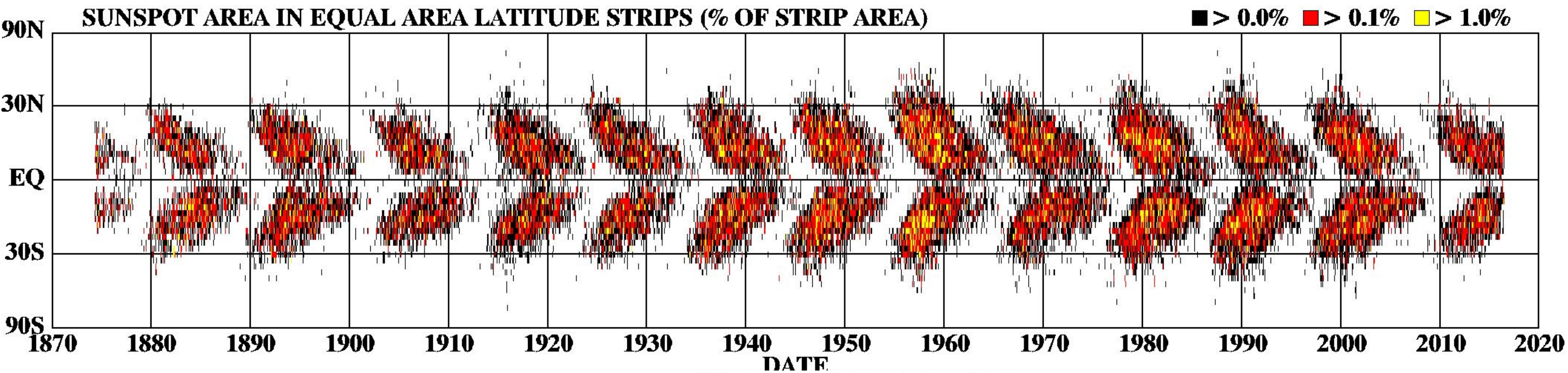
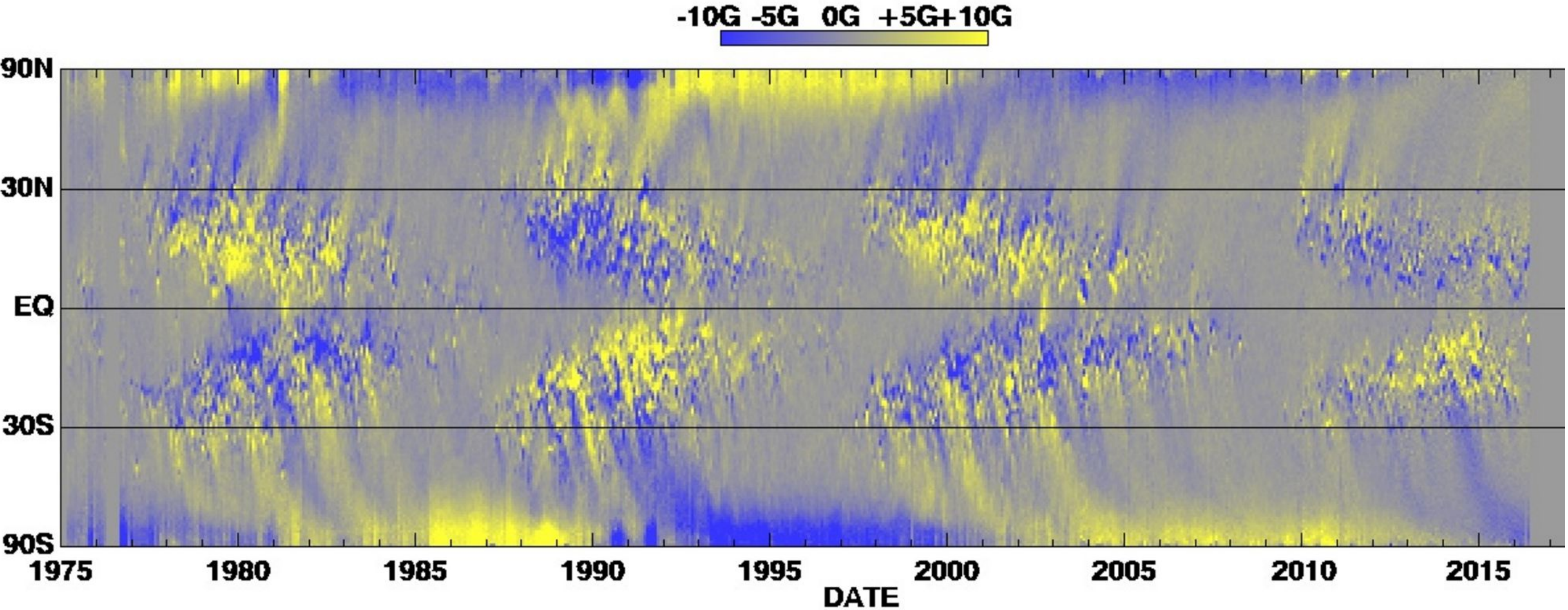
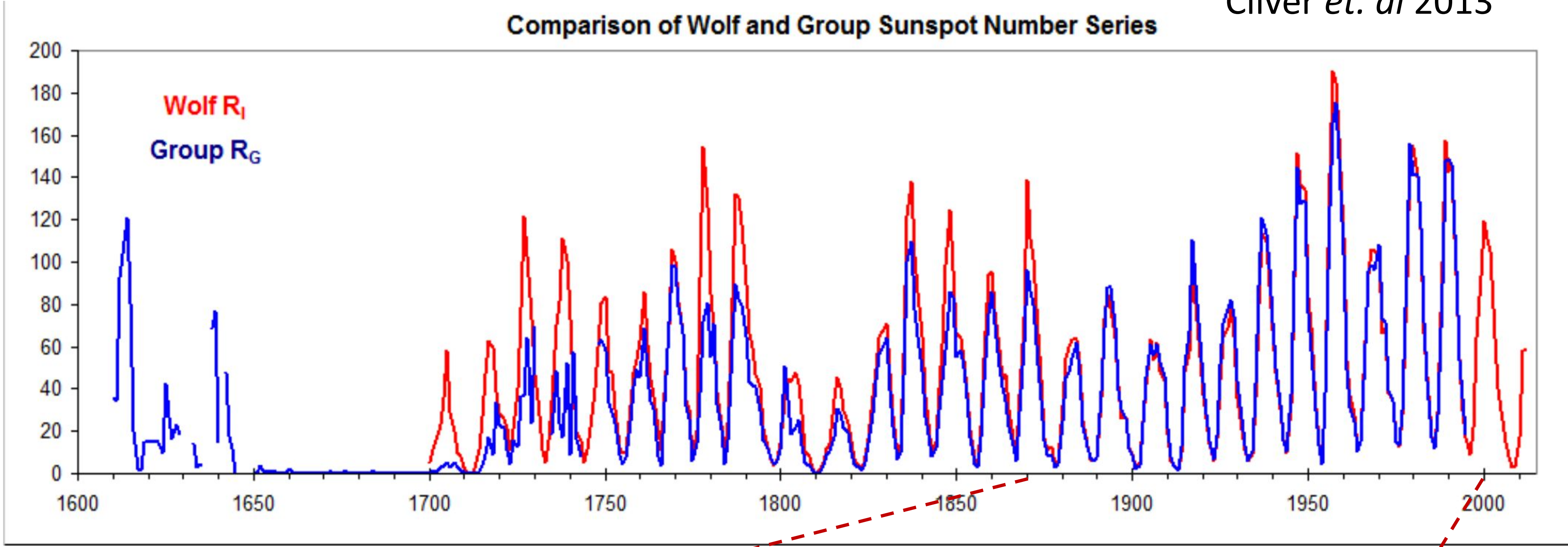


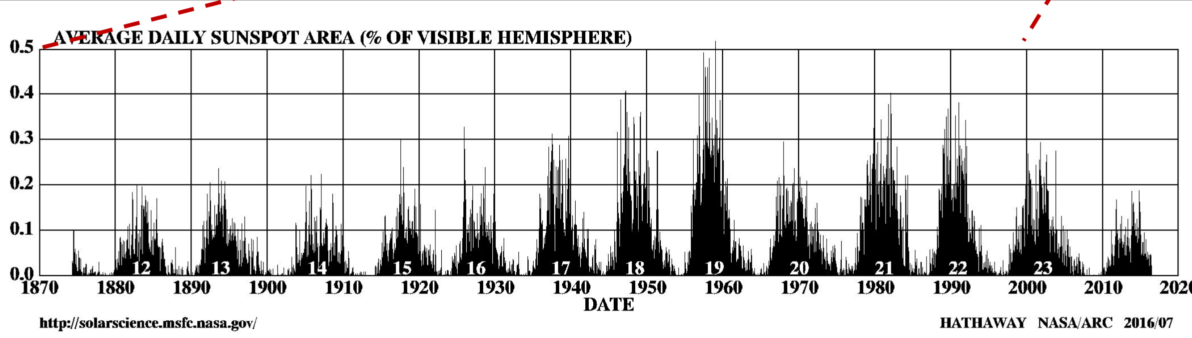


Hathaway NASA ARC 2016/07

Long(er)-term variation on the solar dynamo:

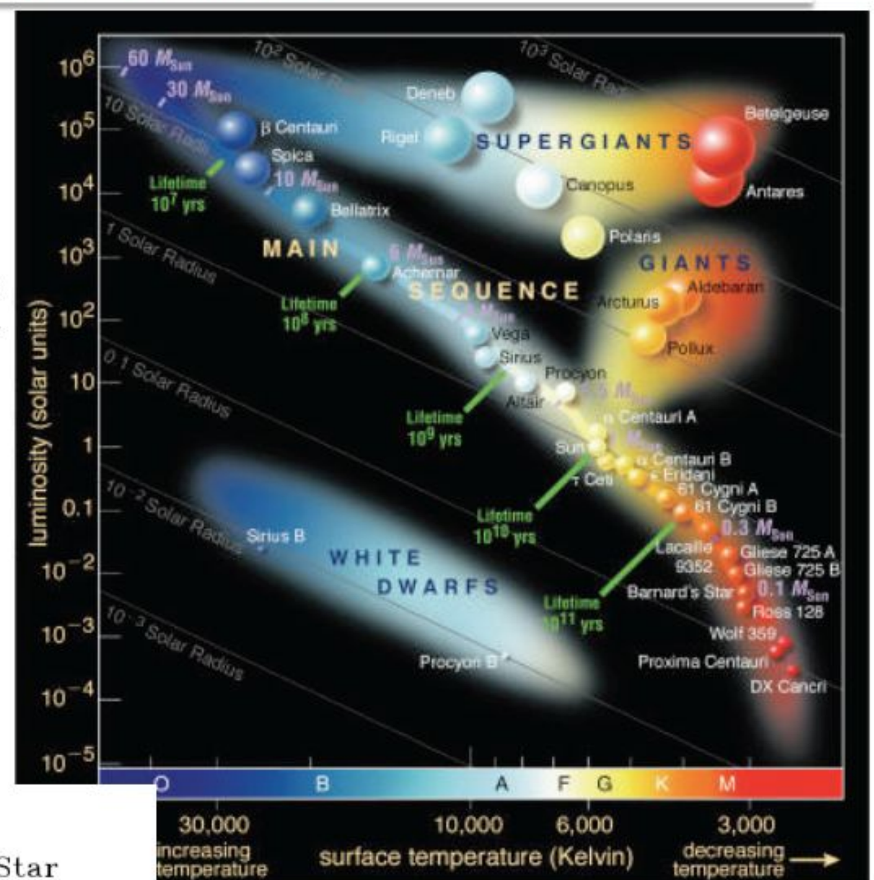
Cliver *et. al* 2013

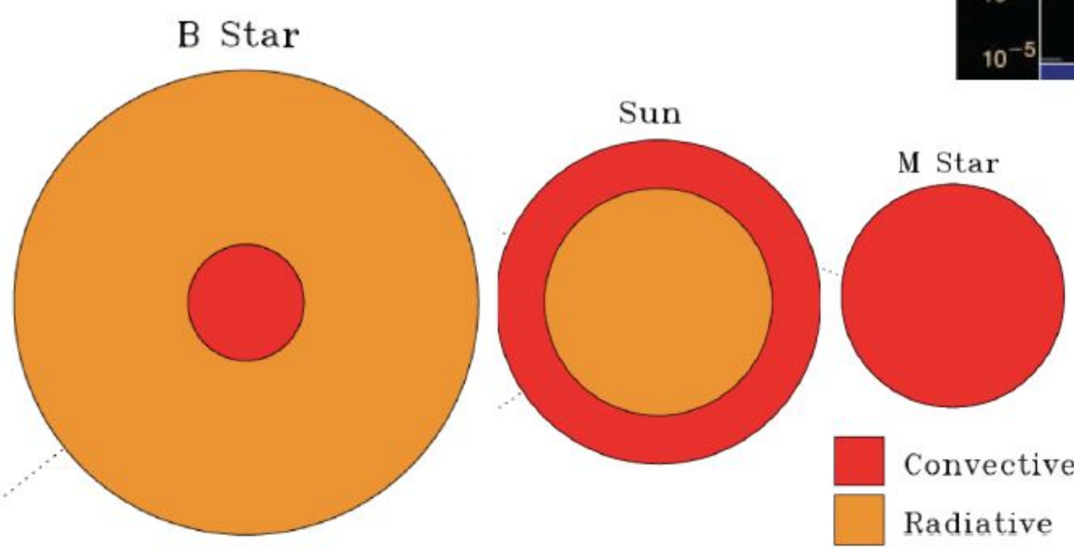




Stellar Magnetic Fields

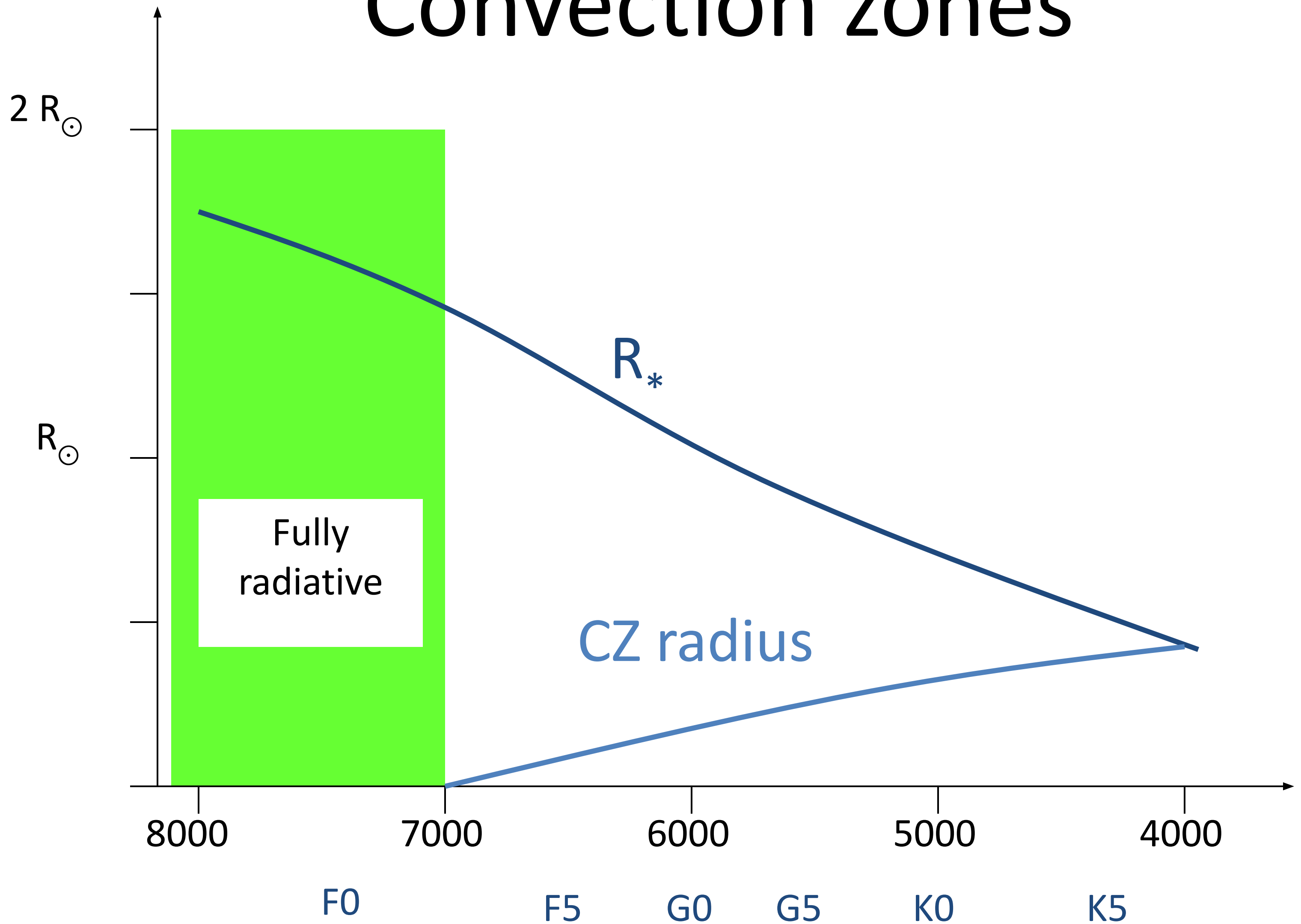
- correlations between stellar types and magnetic field properties, probably due to geometry of convection zones
- stars with outer convection zones (late-type stars) have observed magnetic fields whose strength tends to increase with their angular velocity
- Cyclic variations are known to exist only for spectral types between G0 and K7).





Stanley

Convection zones

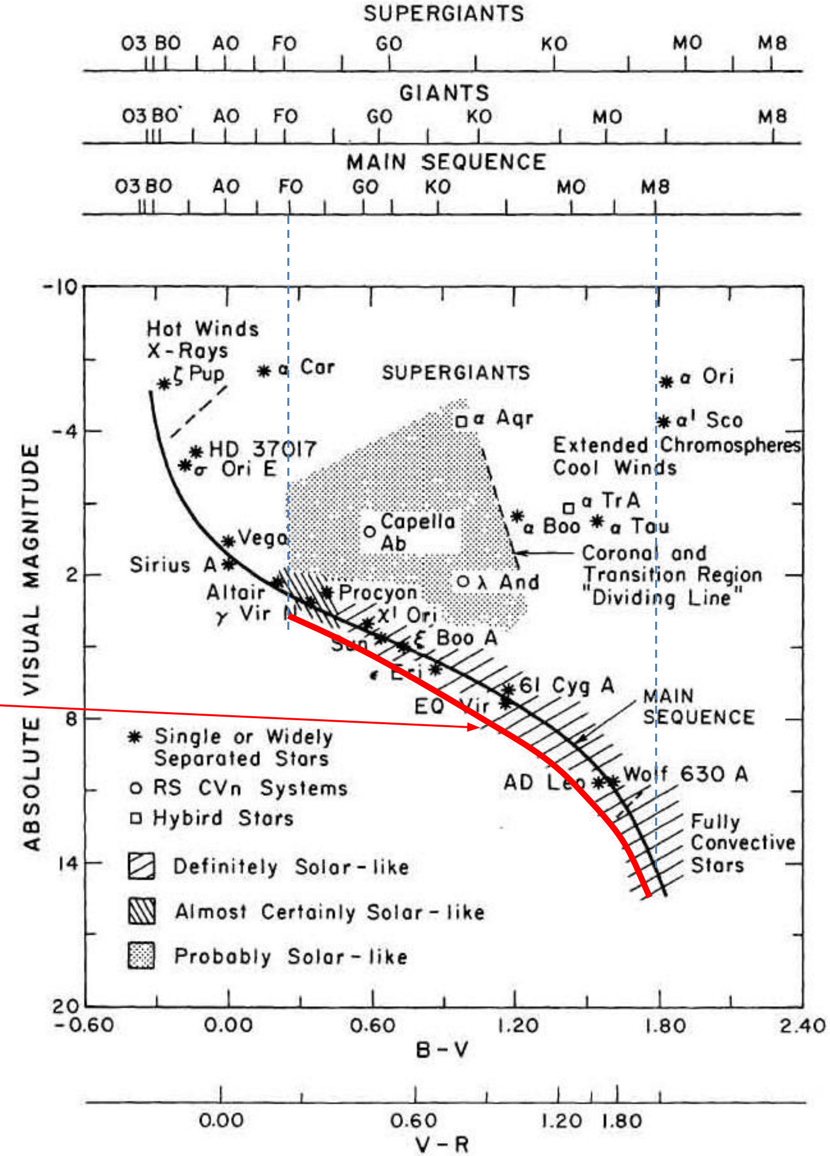


Other stars

Evidence of magnetic activity

Activity on main sequence: types **F M**

B-V > 0.4



(From Linsky 1985)

Activity 50: Relate the Rossby number in Eq. (4.2) to the dynamo number C_Ω and the magnetic Reynolds number R_m in Eq. (4.20):
$N_R = R_m^{\,2} / C_\Omega$.

$$N_{\rm R} = \frac{v_{\rm t}}{\Omega L_{\rm t}}\,, \qquad (4.2)$$

$$C_\alpha = \frac{\alpha_{\rm t} R}{\eta}\,, \qquad C_\Omega = \frac{\Omega_{\rm t} R^2}{\eta}\,, \qquad \mathcal{R}_{\rm m} = \frac{u_{\rm t} R}{\eta}\,, \qquad (4.20)$$

[redacted]

The Dynamo Number

Dynamo is linear instability for D > D_{crit}

$$D \equiv C_\alpha \times C_\Omega = \frac{\alpha_t \Omega_t R^3}{\eta_{CZ}^2} \,.$$

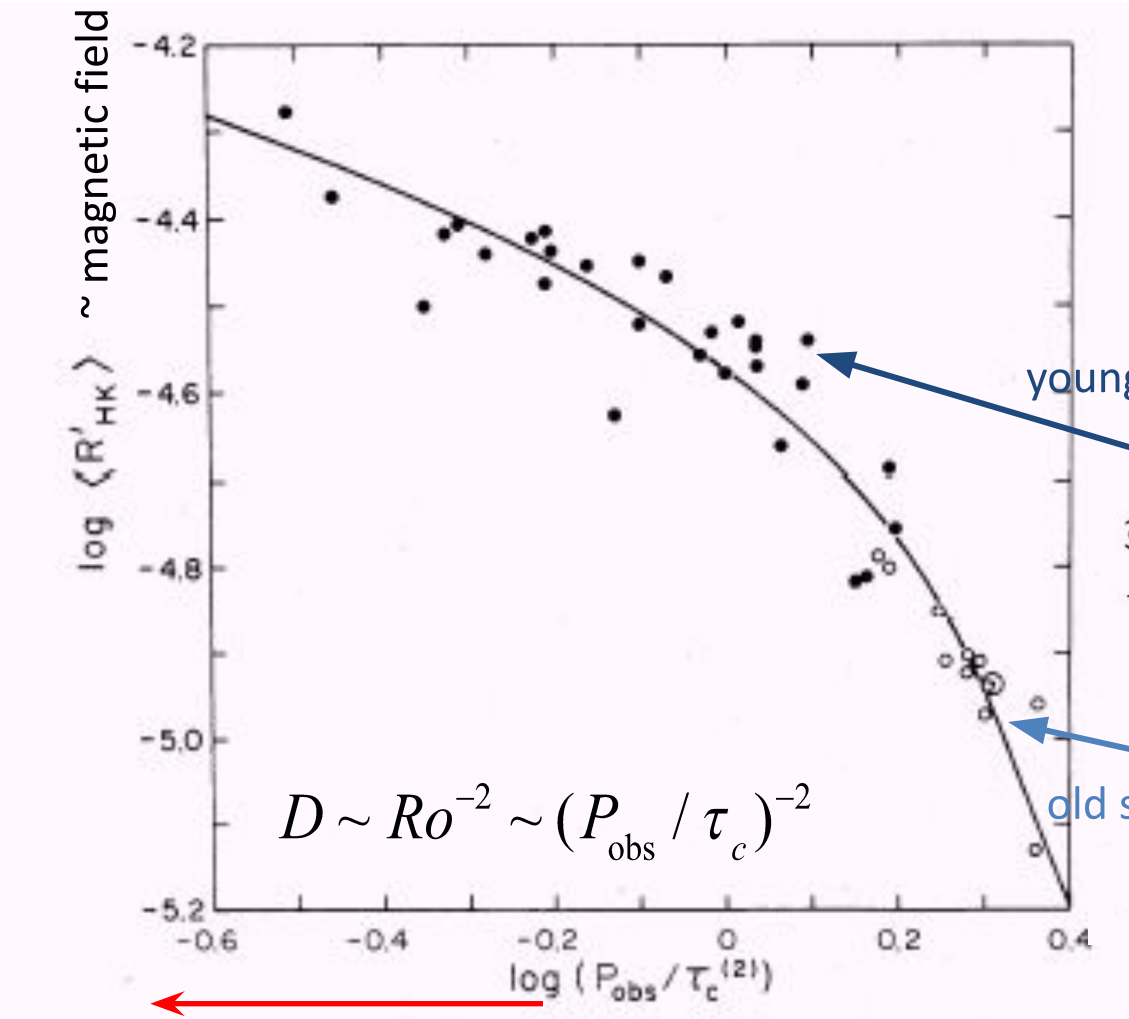
[redacted]

$$C_\alpha = \frac{\alpha_t R}{\eta} \,, \qquad C_\Omega = \frac{\Omega_t R^2}{\eta} \,,$$

$$\eta_{CZ} \sim \frac{R^2}{\tau_c} \qquad \alpha_t \equiv \tau \left\langle \mathbf{v} \cdot \left(\nabla \times \mathbf{v} \right) \right\rangle \sim \Omega_t R$$

$$\boxed{D \sim (\Omega_t \tau_c)^2 \sim Ro^{-2}}$$

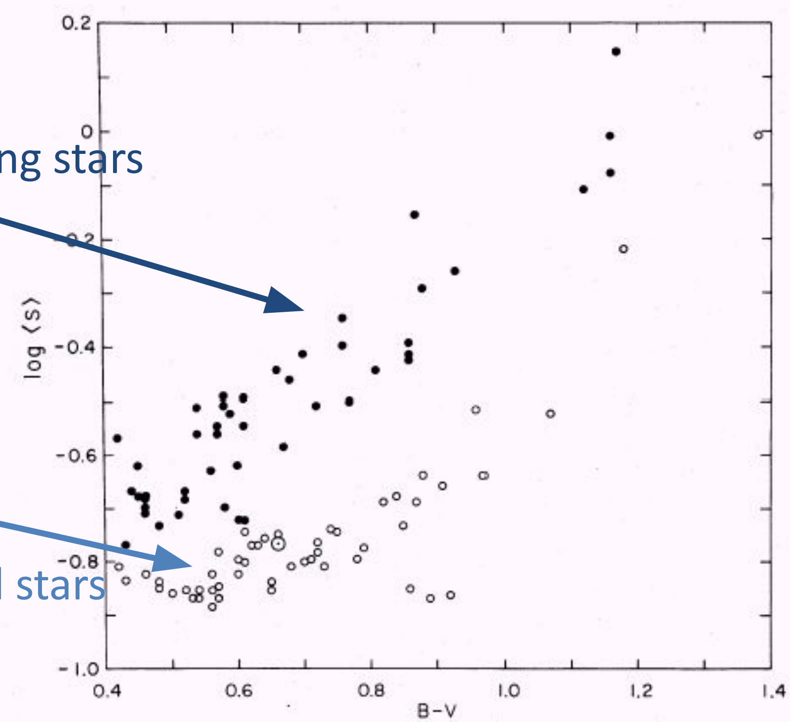
Activity vs. Rossby Number



Stars spin down over time:

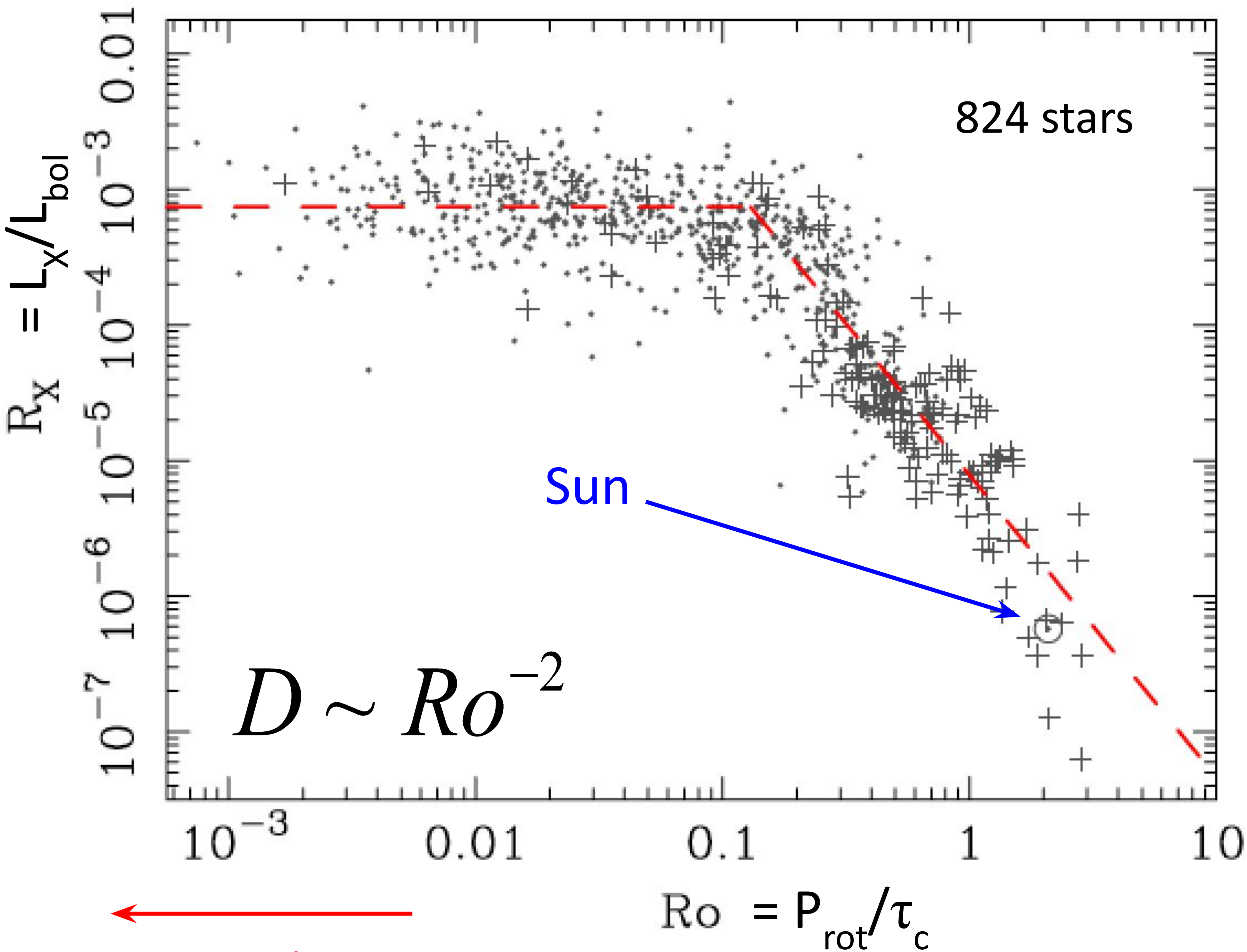
$$\Omega_{\text{rot}} \propto t^{-1/2}. \qquad (10.3)$$

(Skumanich law)



Increasing D

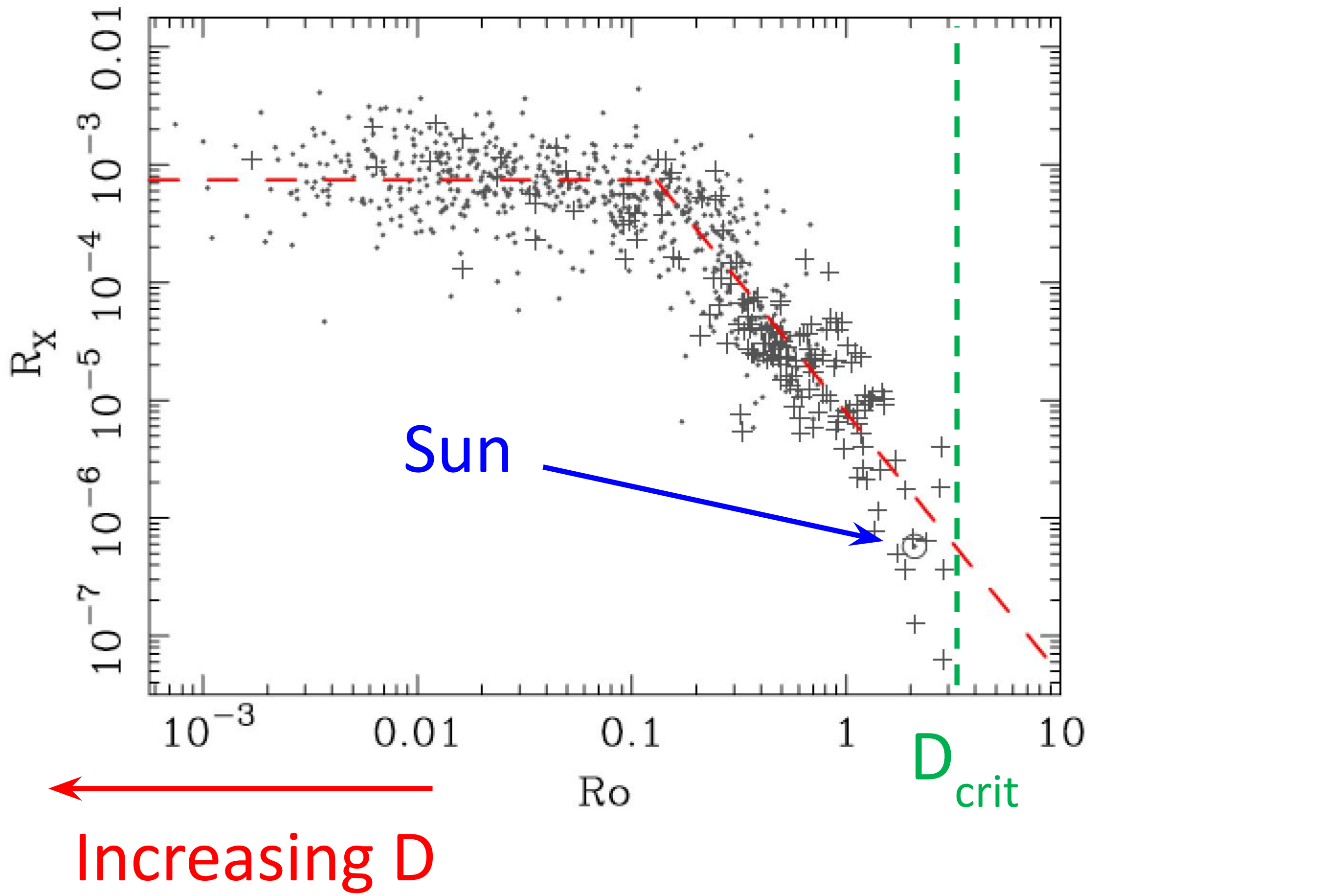
(from Noyes et al. 1984)



Wright et al. 2011

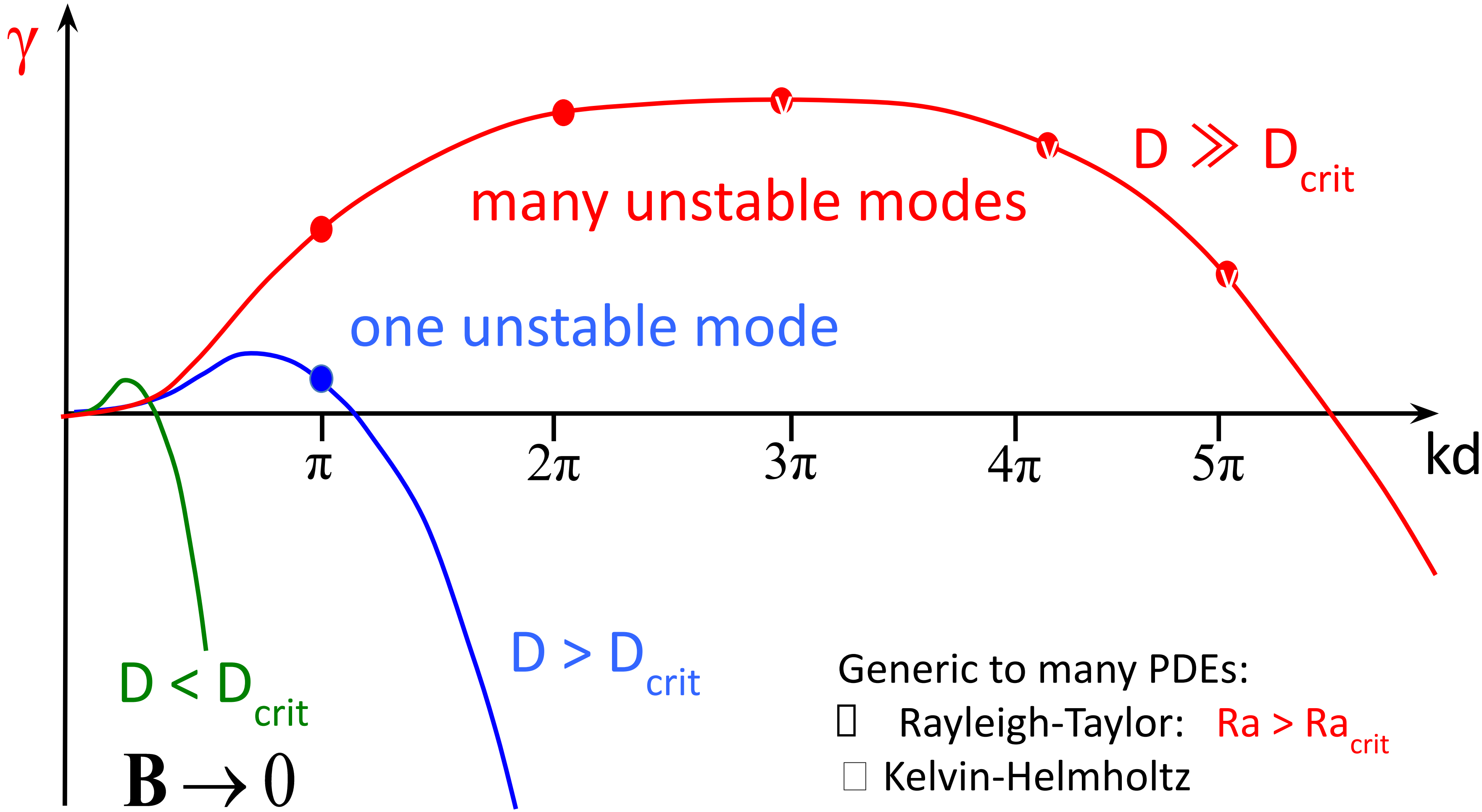
From Cameron & Schussler 2017:

- The solar dynamo is operating near $D \simeq D_{crit}$
- Only single normal mode is unstable
- Long term variation from fluctuations



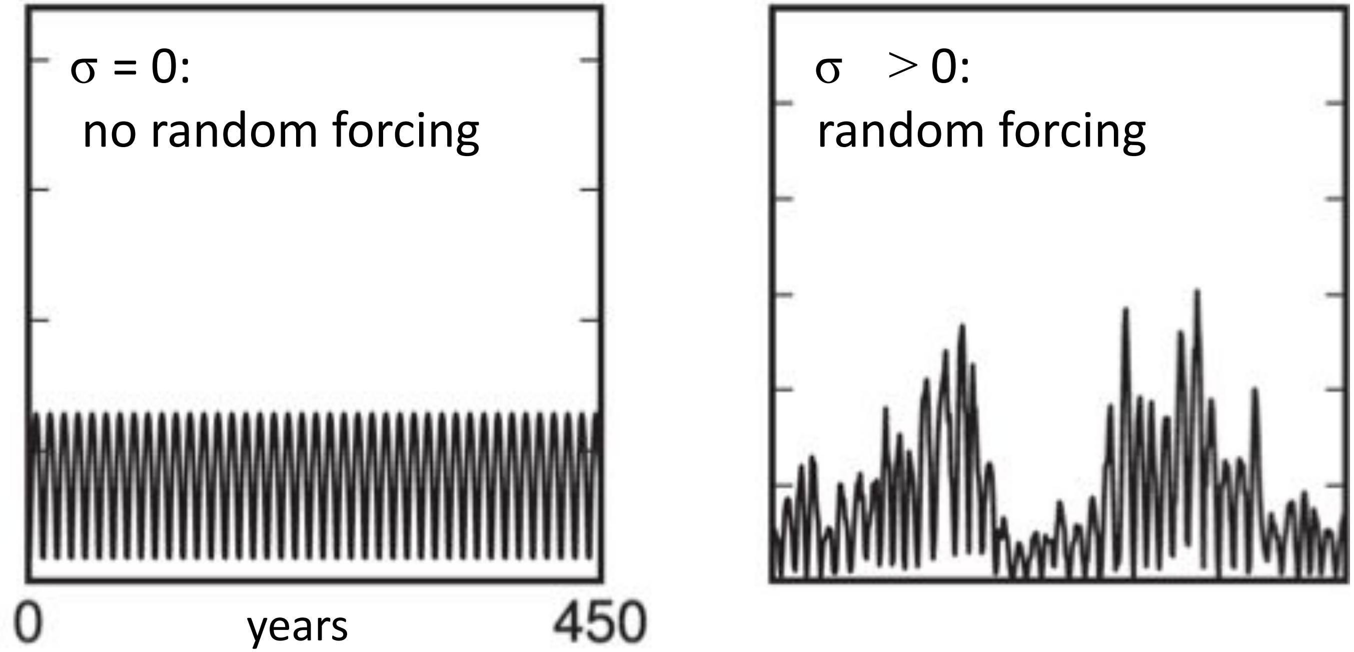
$\gamma > 0$

$$D = \frac{\alpha \Omega d^3}{\eta^2} > 2(kd)^3 \geq 2\pi^3 = D_{\text{crit}}$$



Generic to many PDEs:

- Rayleigh-Taylor: $Ra > Ra_{\text{crit}}$
- Kelvin-Helmholtz

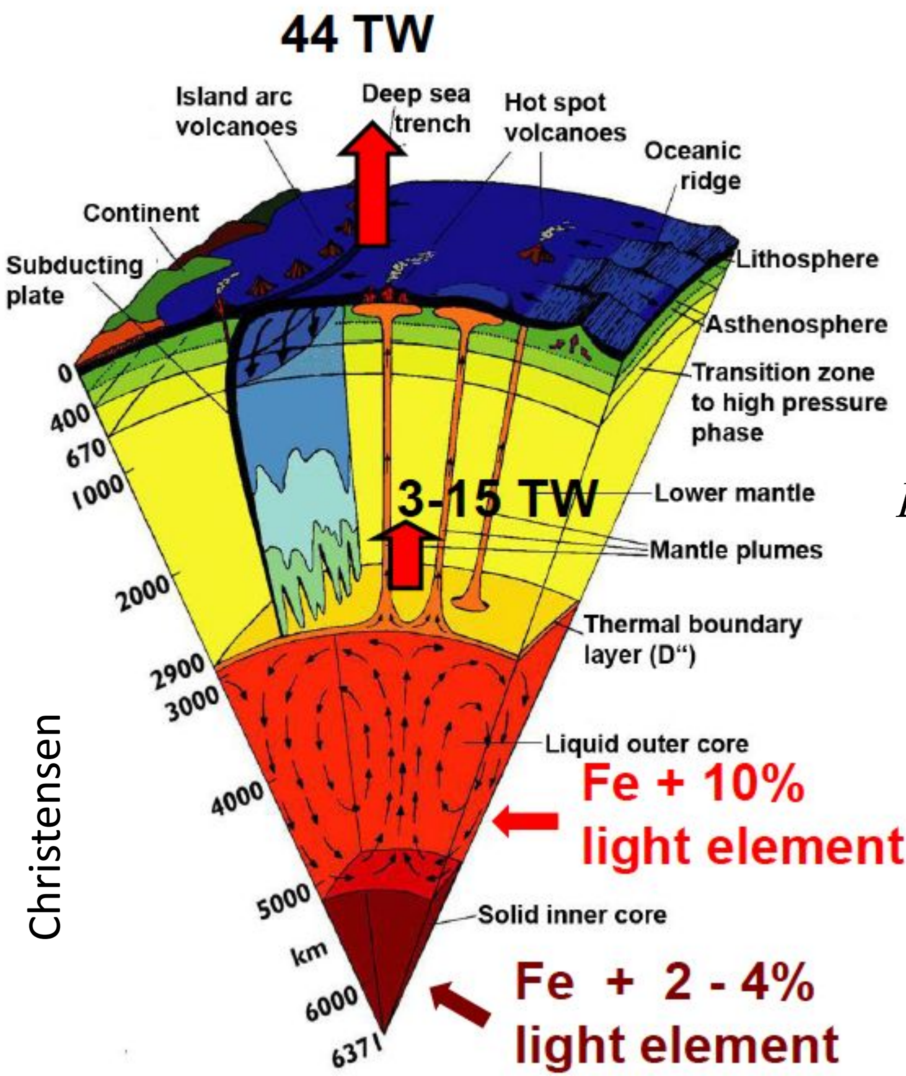


Cameron & Schussler 2017

one normal mode:

$$dX = \underbrace{(\beta + i\omega_0}_{\text{linear instability}} - \overbrace{(\gamma_r + i\gamma_i)|X|^2}^{\text{non-linear saturation}})Xdt + \underbrace{\sigma X dW_c}_{\substack{\text{random forcing} \\ \text{(Wiener process)}}} = 0,$$

How the dynamo works for Earth



Christensen

Non-conducting mantle

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} = 0$$

$$\mathbf{B} = -\nabla \chi \qquad \nabla \cdot \mathbf{B} = -\nabla^2 \chi = 0$$

$$\chi(r,\theta,\phi) = \sum_{\ell,m} g_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+1}$$

$$B_r(r,\theta,\phi) = -\frac{\partial \chi}{\partial r} = \sum_{\ell,m} (\ell+1) g_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+2}$$

simplifies w/ increasing r

Turbulent conducting fluid:

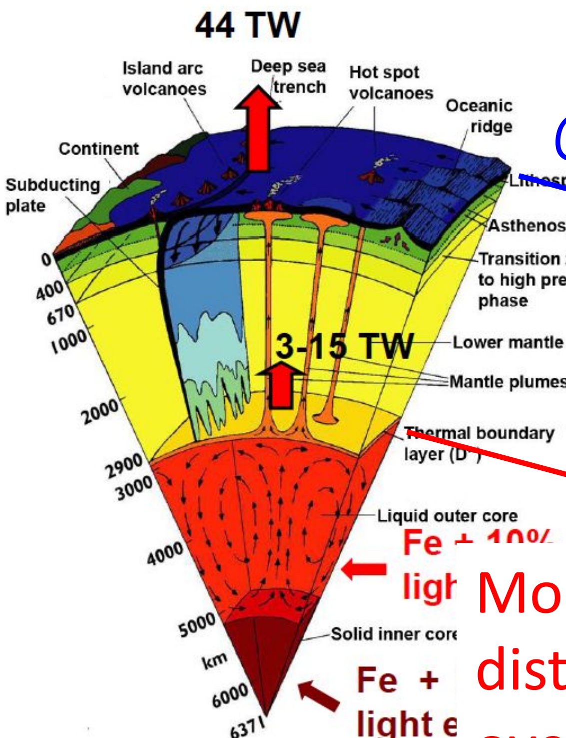
DYNAMO

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \neq 0$$

Complex flows – complex field

$$B_r(r,\theta,\phi) = \sum_{\ell,m} (\ell+1)\tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+2}$$

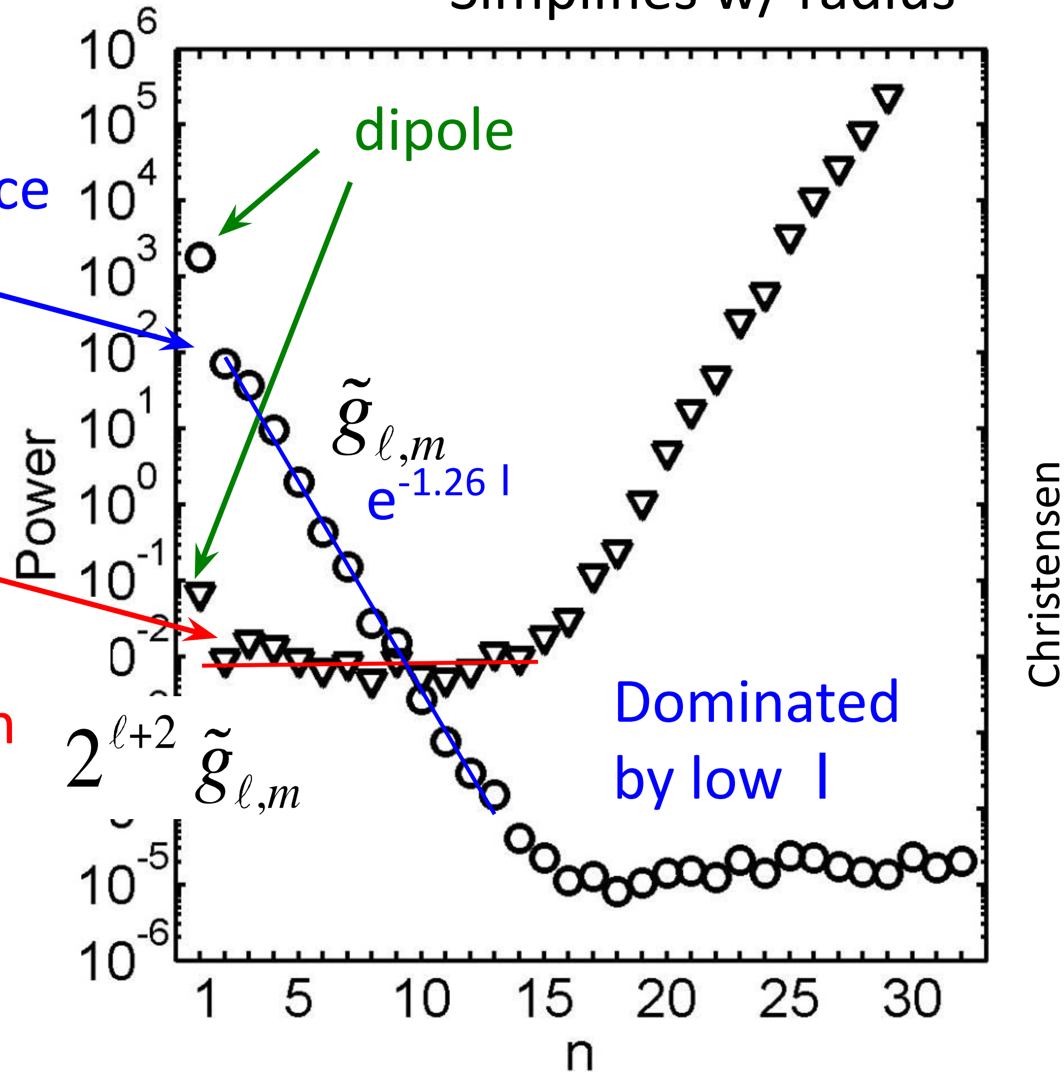
Simplifies w/ radius



@ surface

@ core-mantle boundary

More even distribution over l



Dominated by low l

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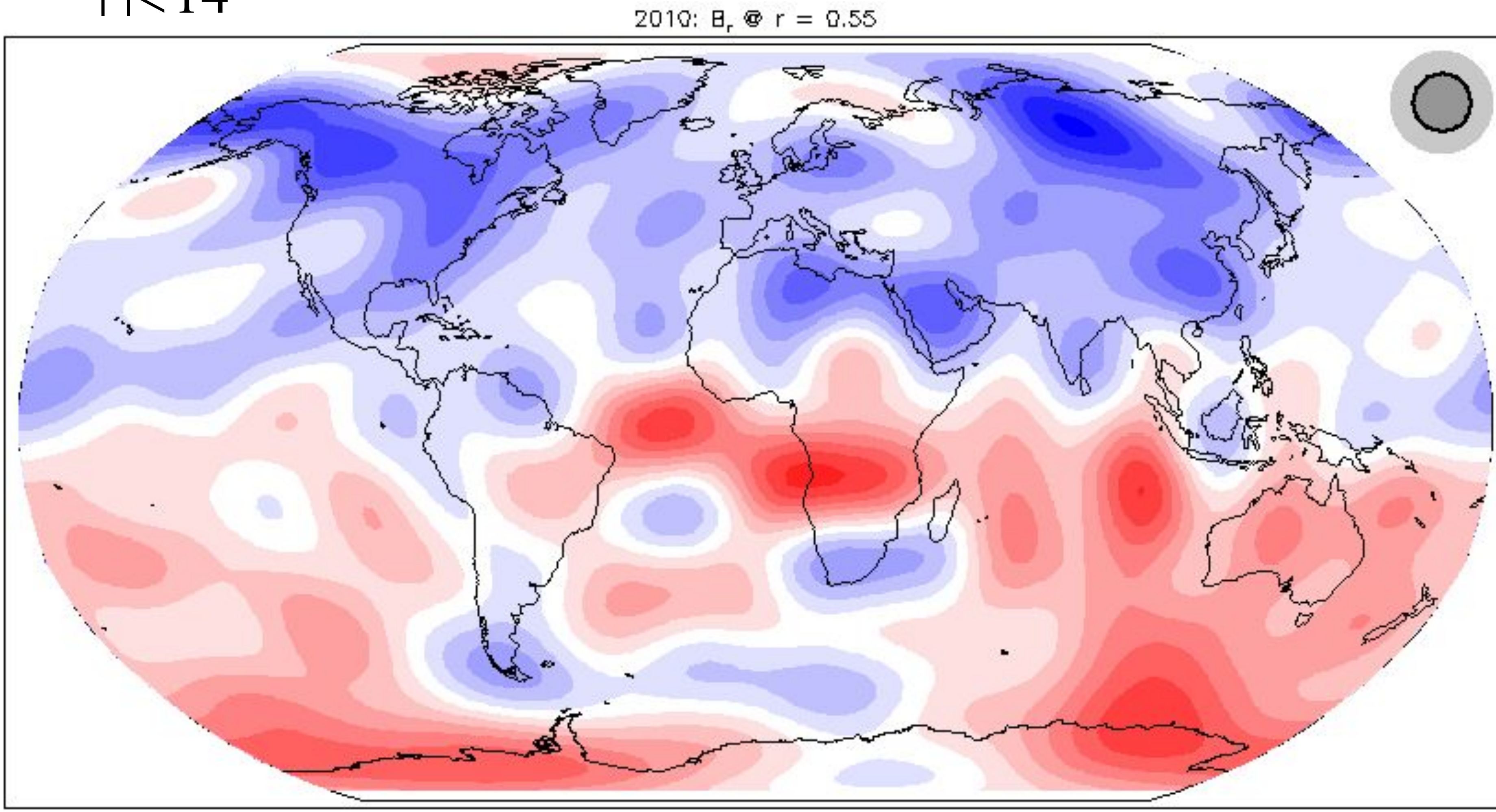
Observation: what B_r looks like today

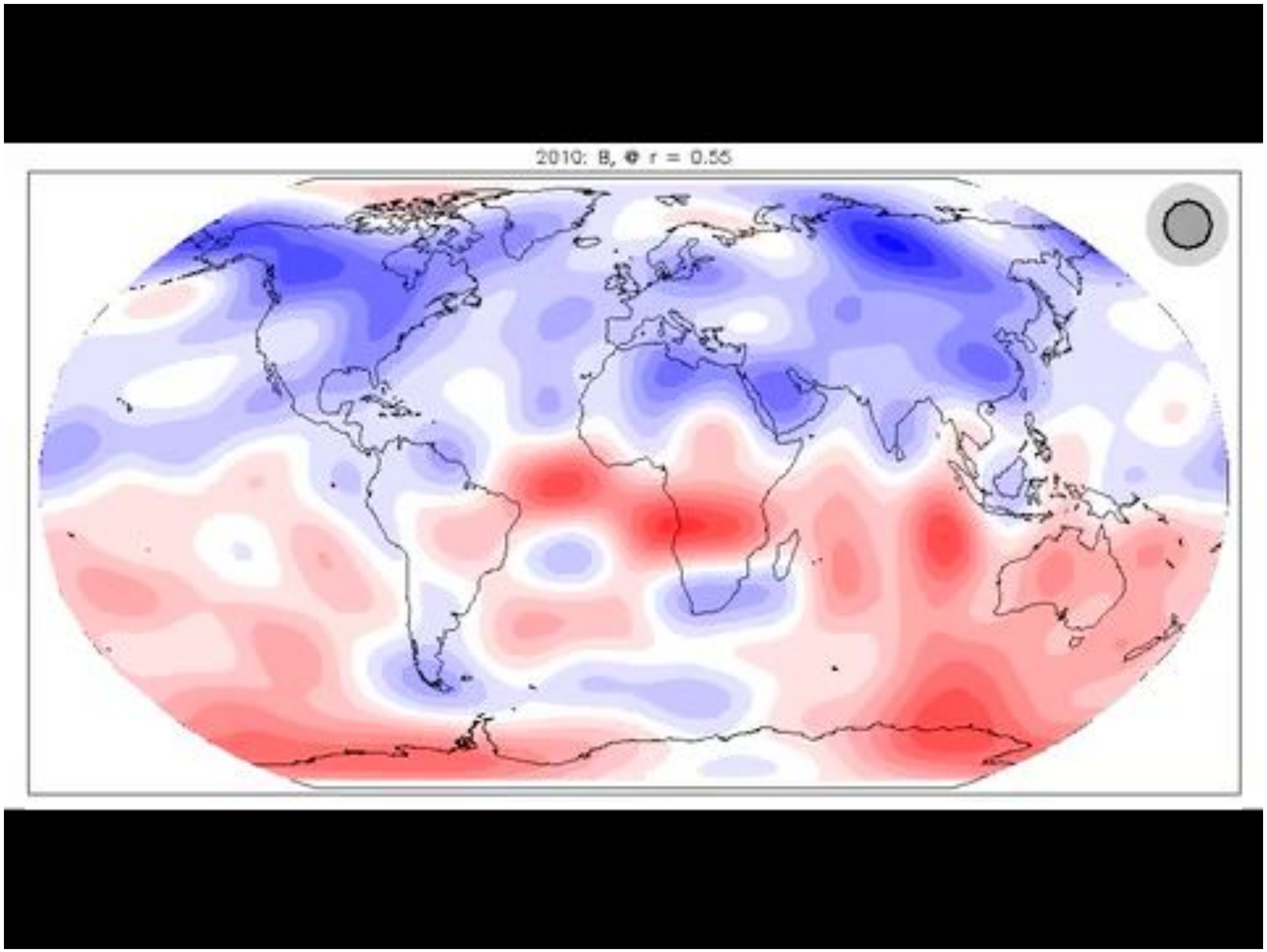
@ core-mantle boundary: lower boundary of potential region

∏< 14



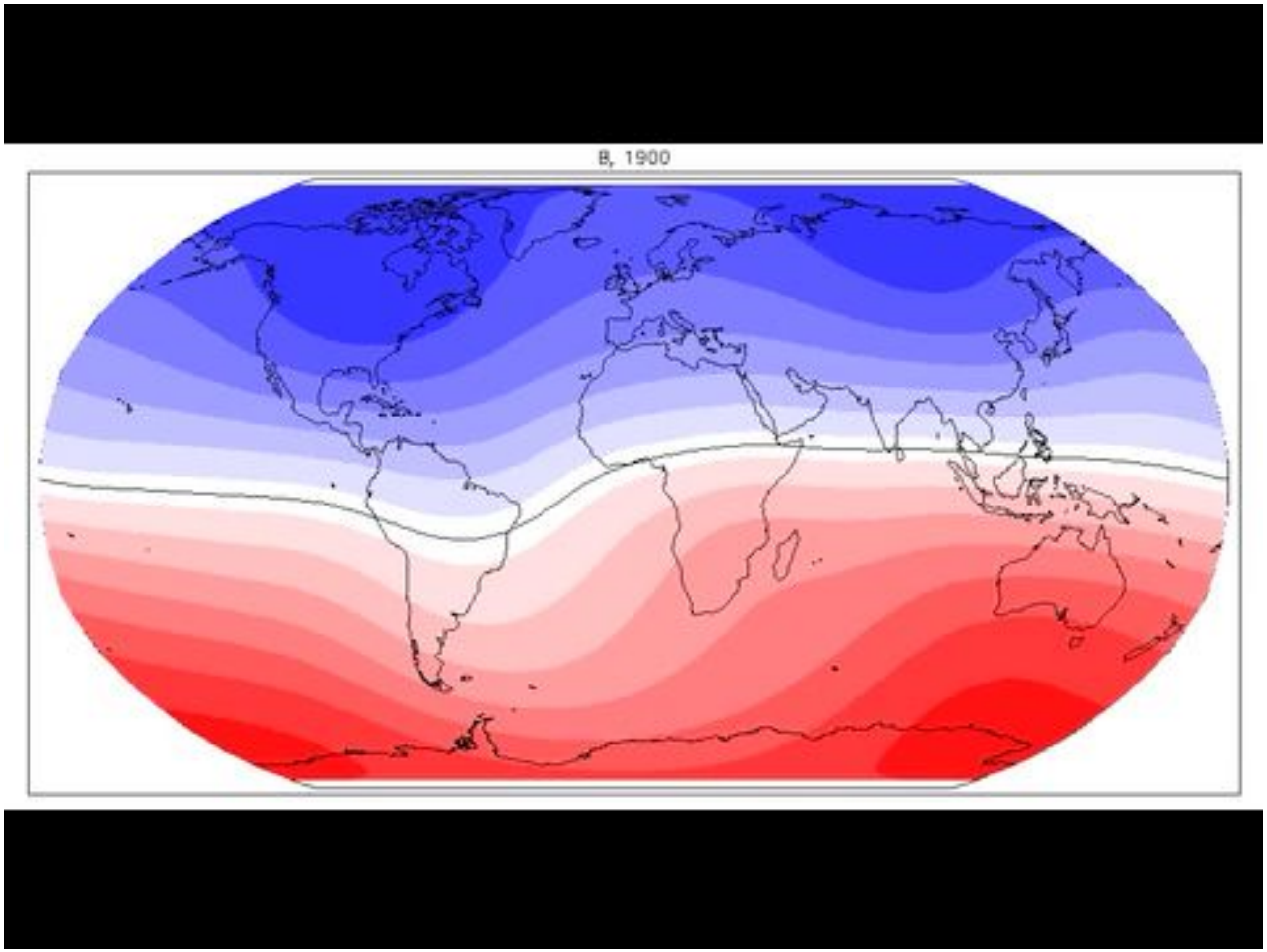
Simplifies w/ increasing r

$$B_r(r,\theta,\phi) = -\frac{\partial \chi}{\partial r} = \sum_{\ell,m} (\ell+1) g_{\ell,m} \, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+2}$$



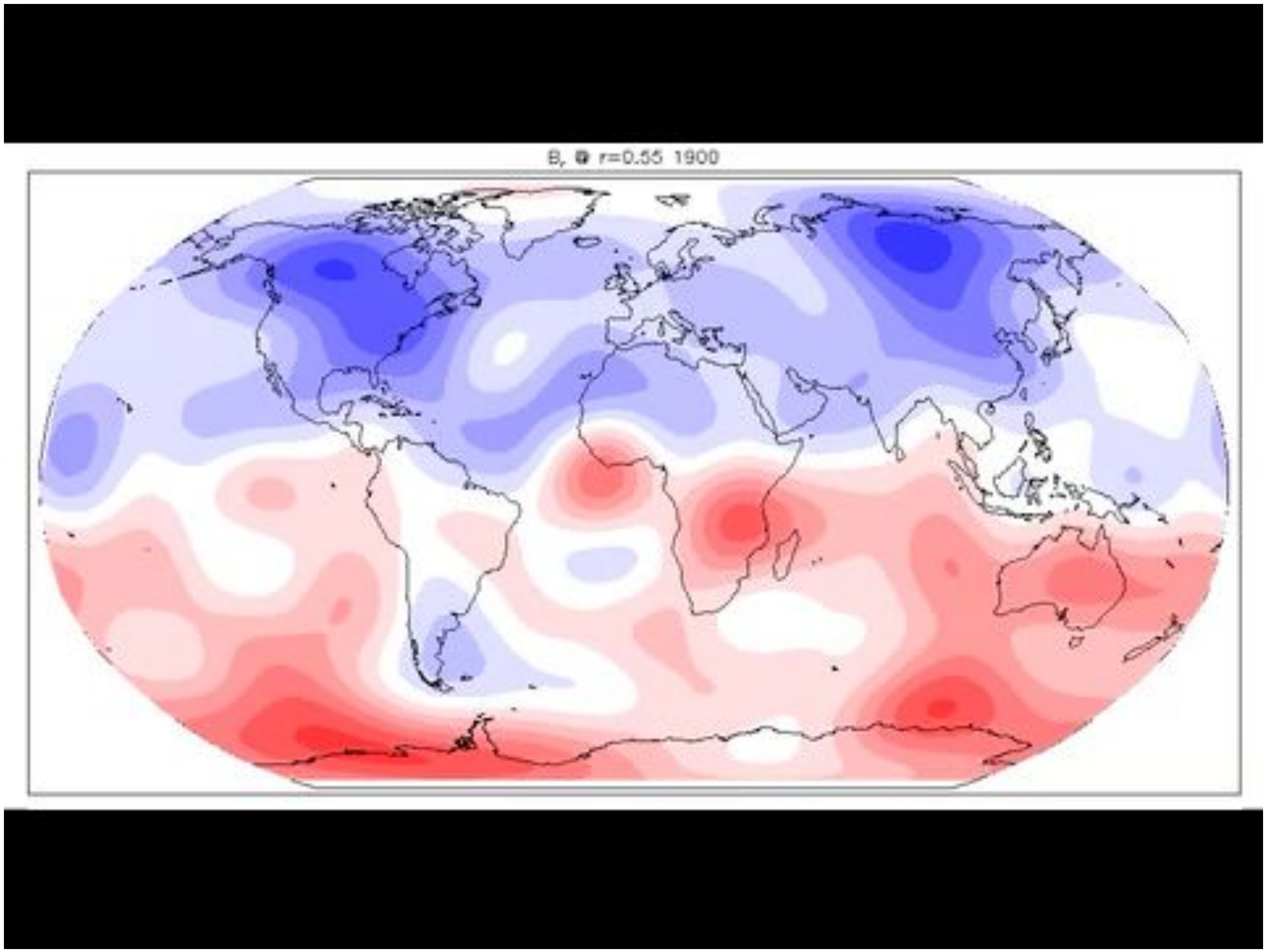
Evolution of field

@ surface for 100 years

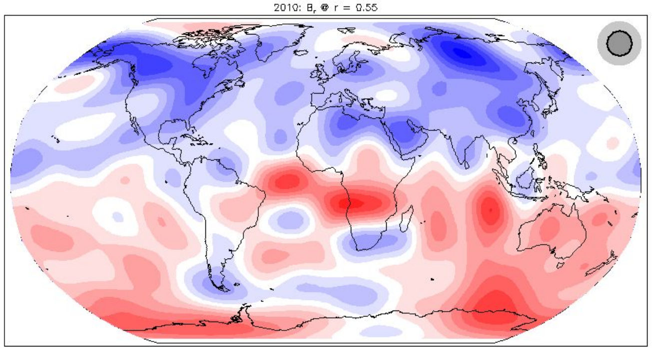


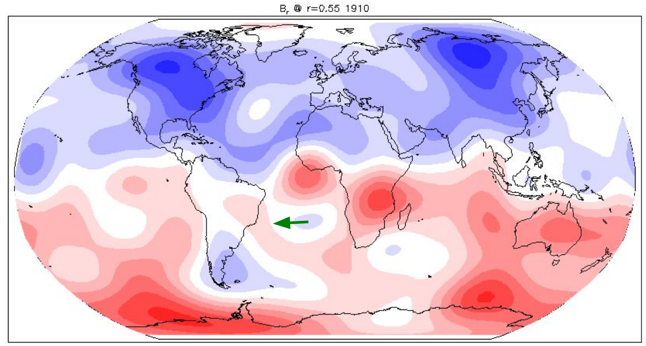
Evolution of field

@ core-mantle boundary for 100 years



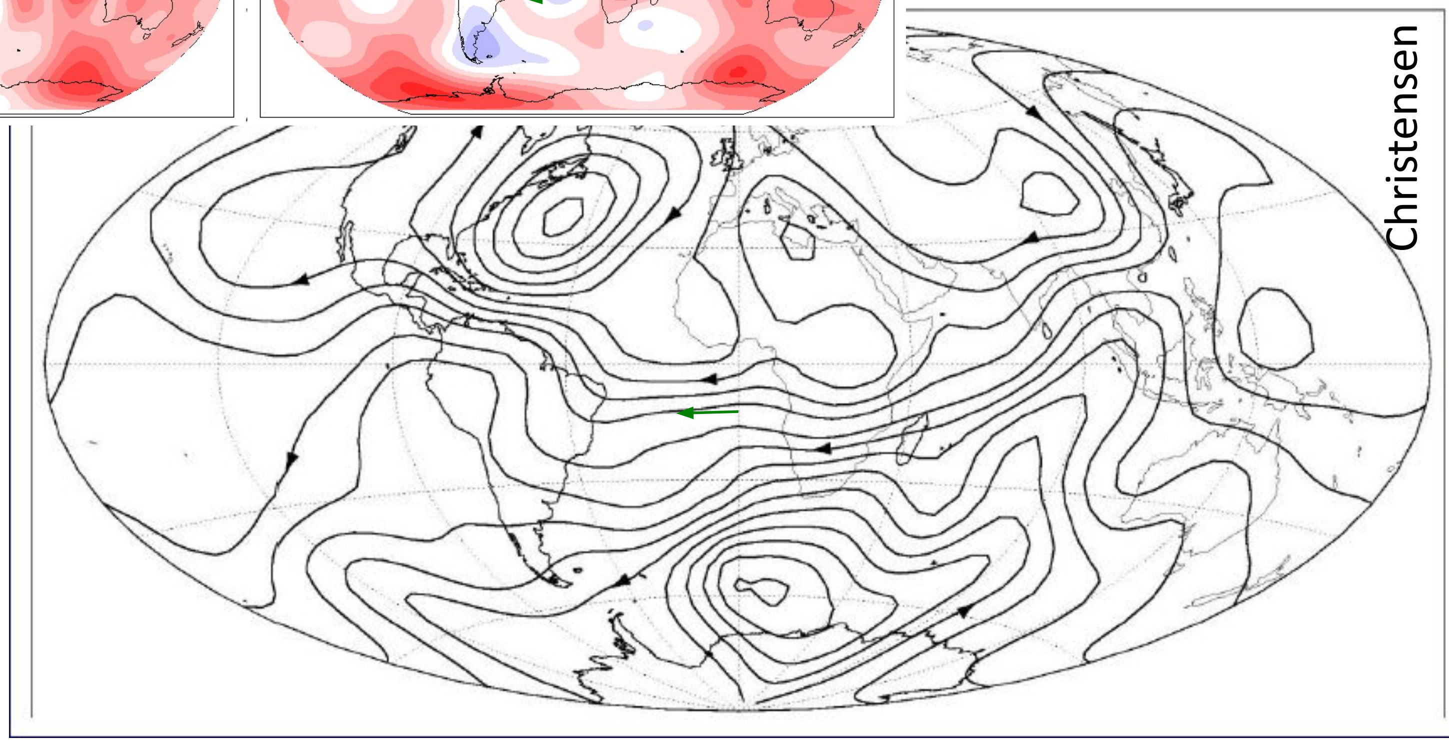
Use evolution to infer fluid velocity





$v \sim 3 \times 10^{-4}$ m/s

- Δx = 1,000 km
- in 100 years



Subsonic flow,
Ignore stratification

$$\nabla \cdot \mathbf{v} = \mathbf{0}$$

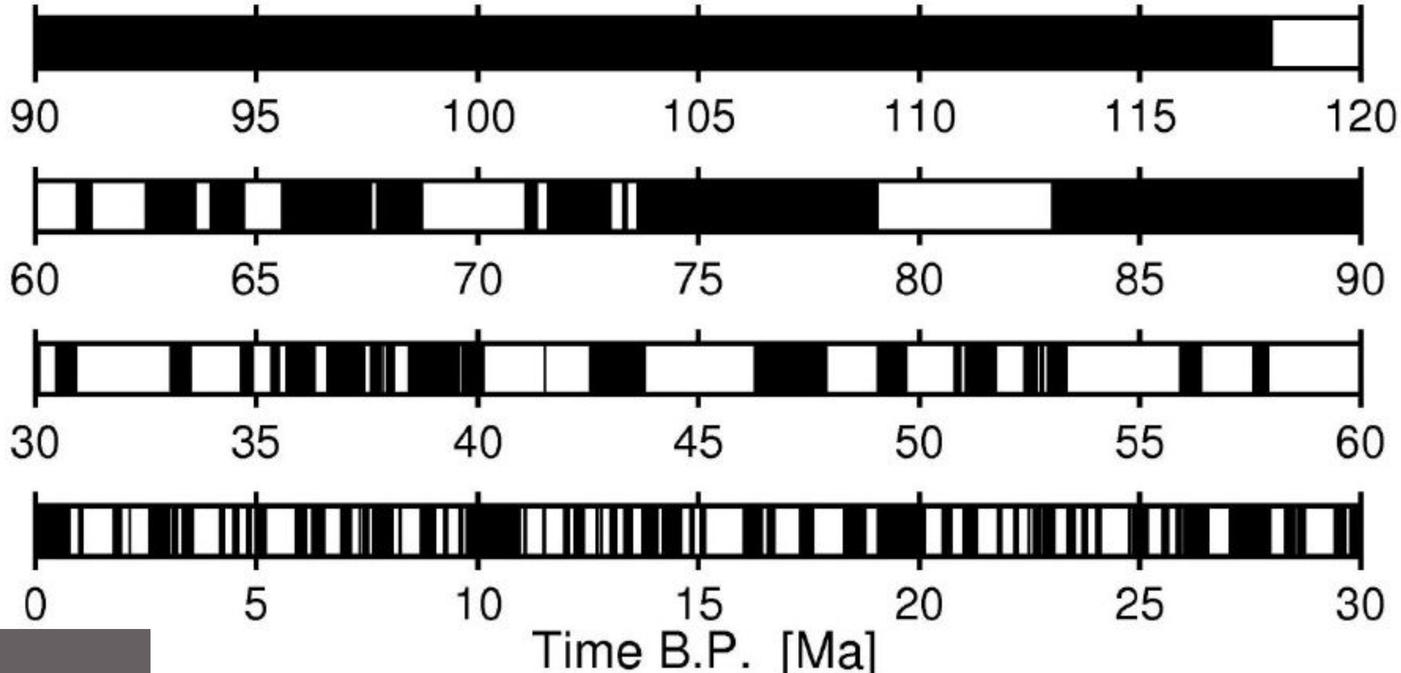
Ignore diffusion

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{v} - \cancel{\mathbf{B}(\nabla \cdot \mathbf{v})} + \cancel{\eta \nabla^2 \mathbf{B}}$$

known

$$\frac{\partial B_r}{\partial t} + \nabla \cdot (\mathbf{v} B_r) = 0$$

Longer-term evolution



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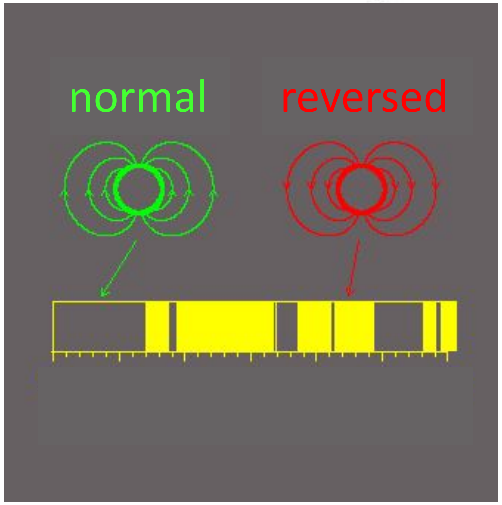
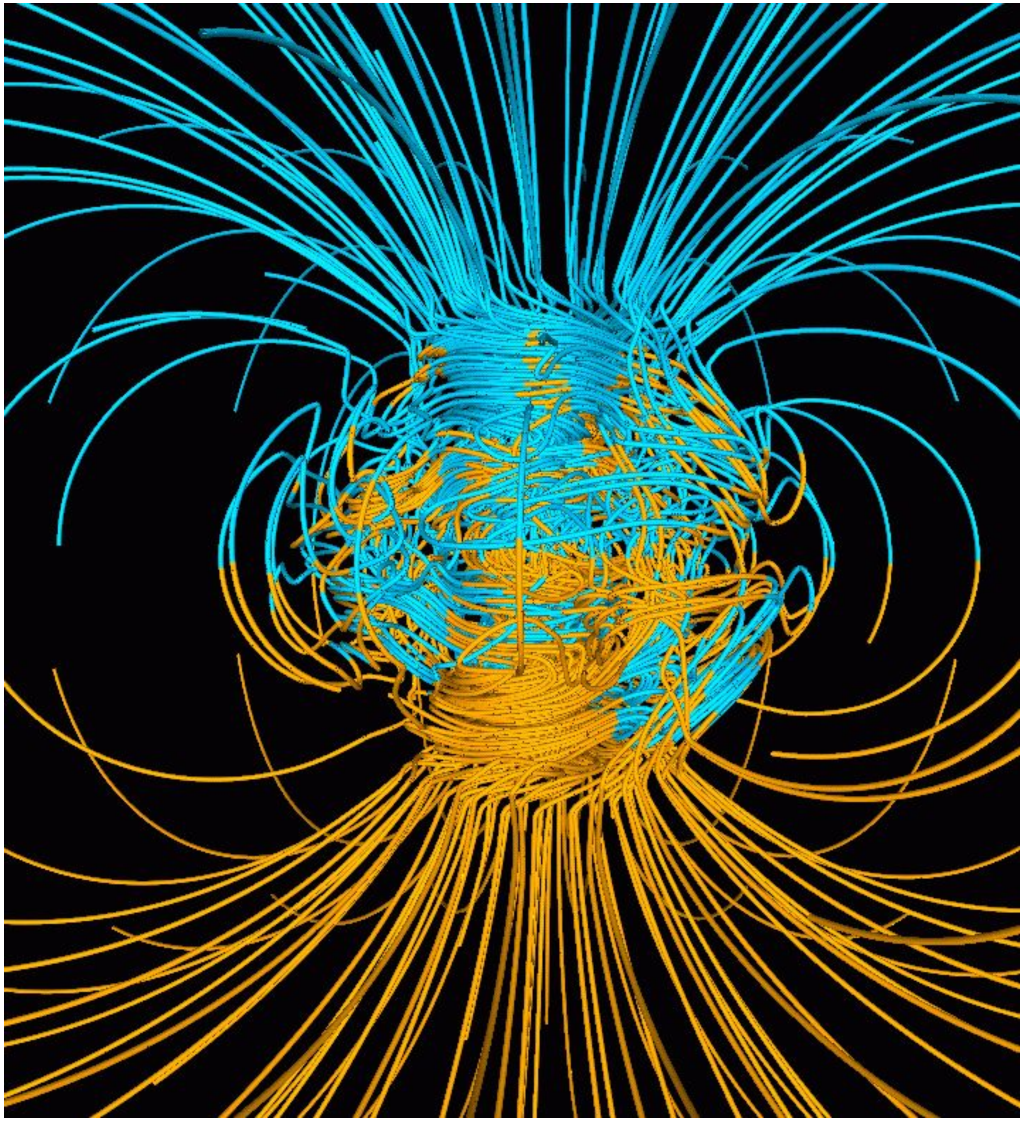


Like toy dynamo, Earth works in 2 modes. Flips between them seemingly at random

Model geodynamo

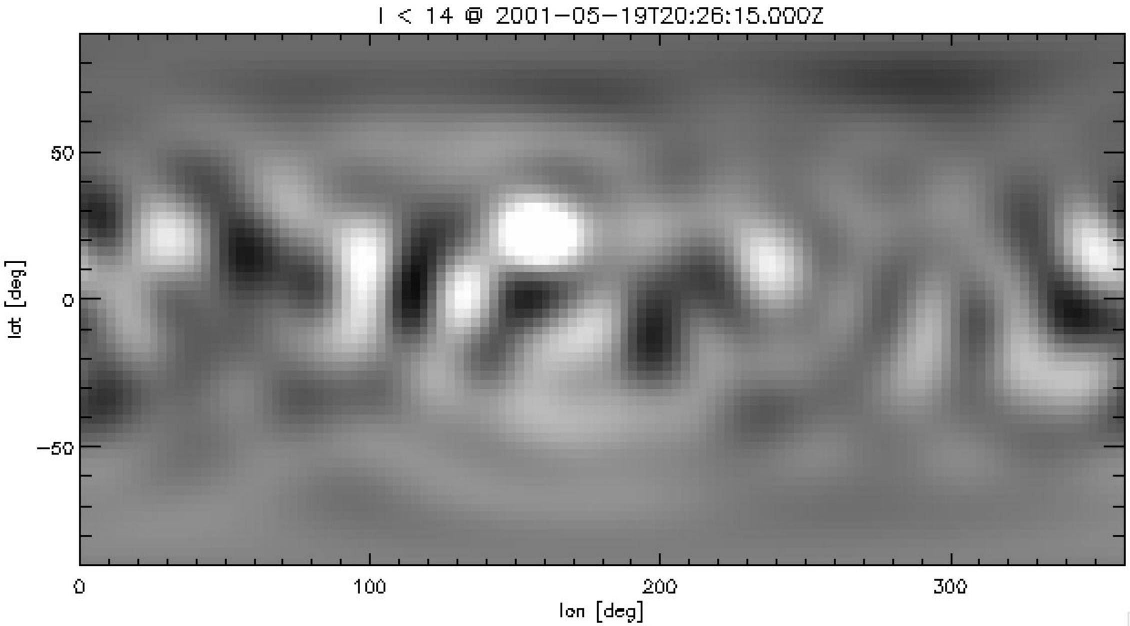
http://www.es.ucsc.edu/~glatz/geodynamo.html

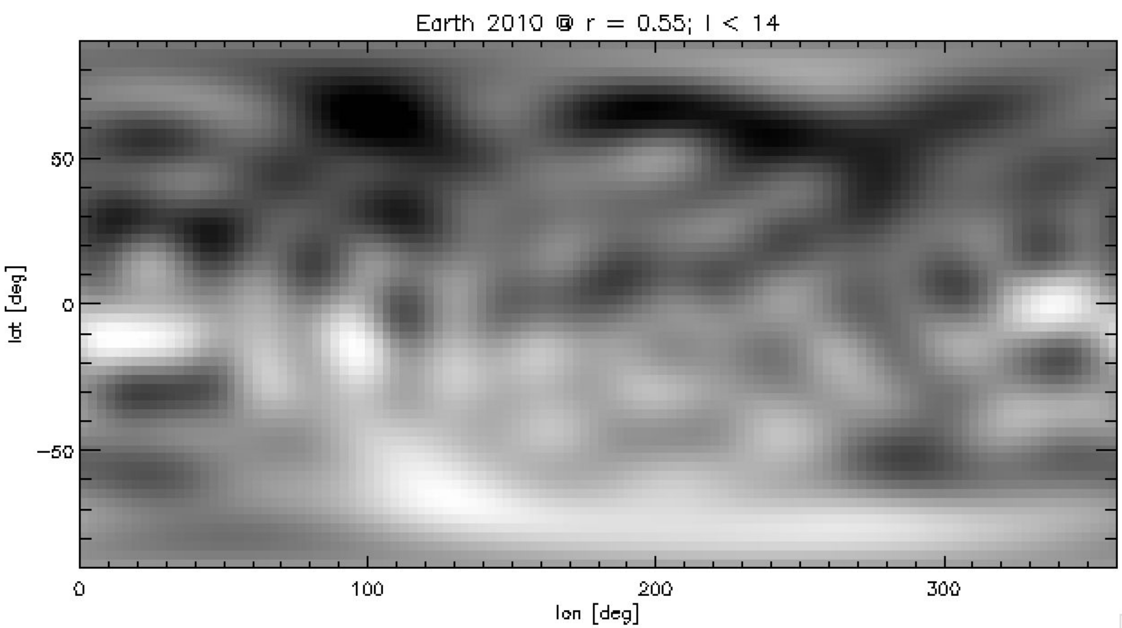


- Glatzmaier & Roberts 1995
- Numerical solution of MHD
- Toroidal structure inside convecting core

Activity 57: Summarize the contrast between the dynamo of a terrestrial planet with that of stars as discussed in this Chapter 4: consider, among others, flow speed, rotation period, stratification, differential rotation, and meridional advection.

	η [m^2/s]	υ [m^2/s]	L [m]	v [m/s]	Ω [rad/s]	Rm	Re	Ro
Sun (CZ)	1	10^{-2}	10^{8}	1	10^{-6}	10^{8}	10^{10}	10^{-2}
Earth (core)	1	10^{-5}	10^{6}	10^{-4}	10^{-4}	10^{2}	10^{7}	10^{-6}





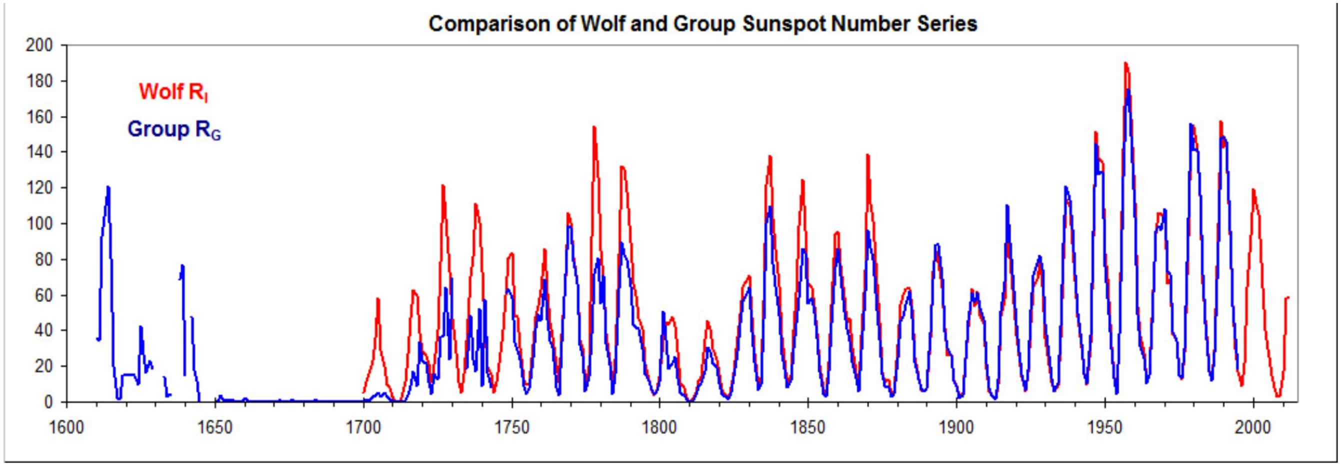
$Rm = 10^8$
$Ro = 10^{-2}$

Dynamo comparison: Sun vs. Earth

$Rm = 10^2$
$Ro = 10^{-6}$

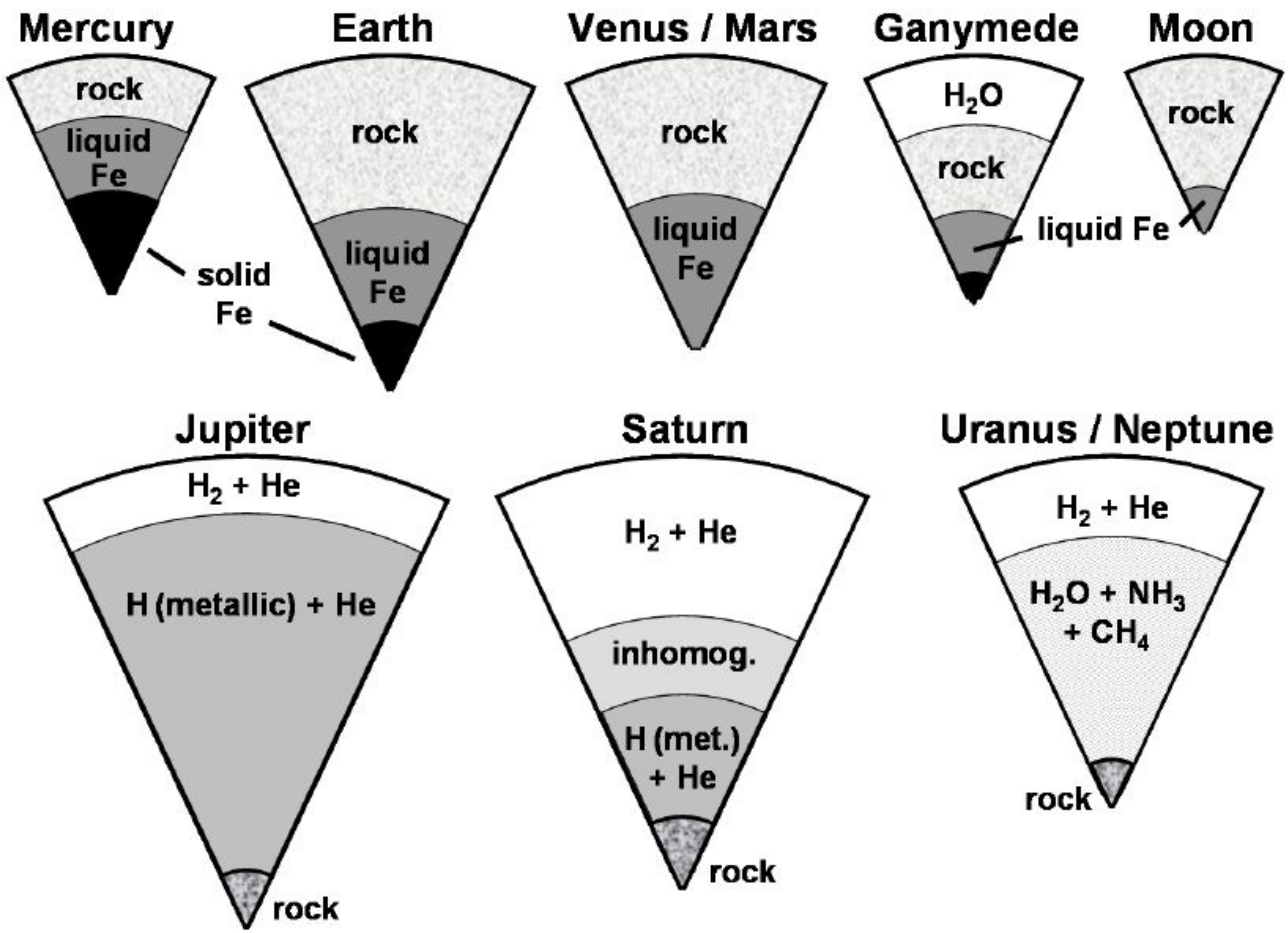
$$D \sim Ro^{-2}$$

$Ro = 10^{-2}$ $D \sim 10^{4} \gtrsim D_{crit}$



$Ro = 10^{-6}$ $D \sim 10^{12} \gg D_{crit}$

Other planets



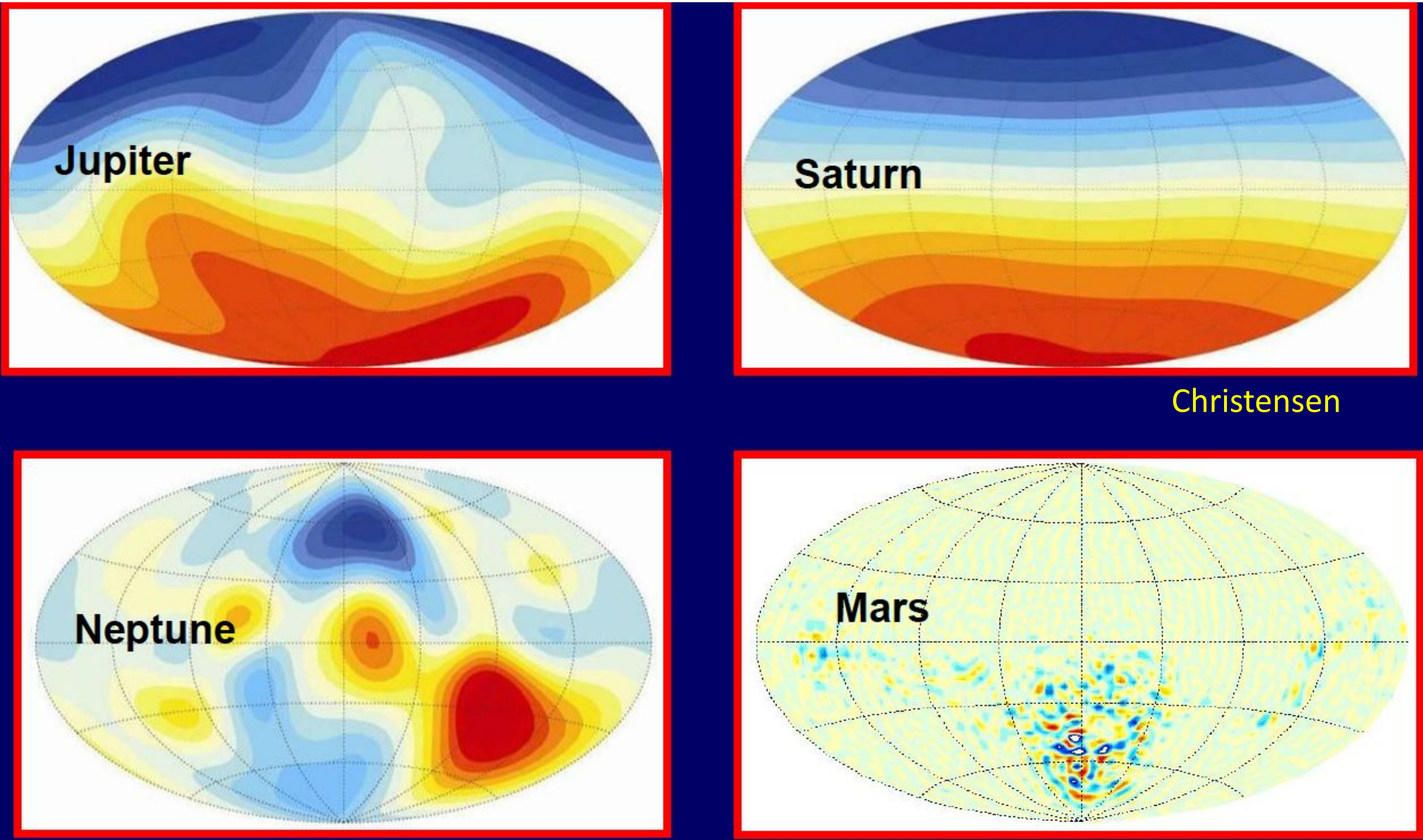
Christensen

Other planets

Christensen

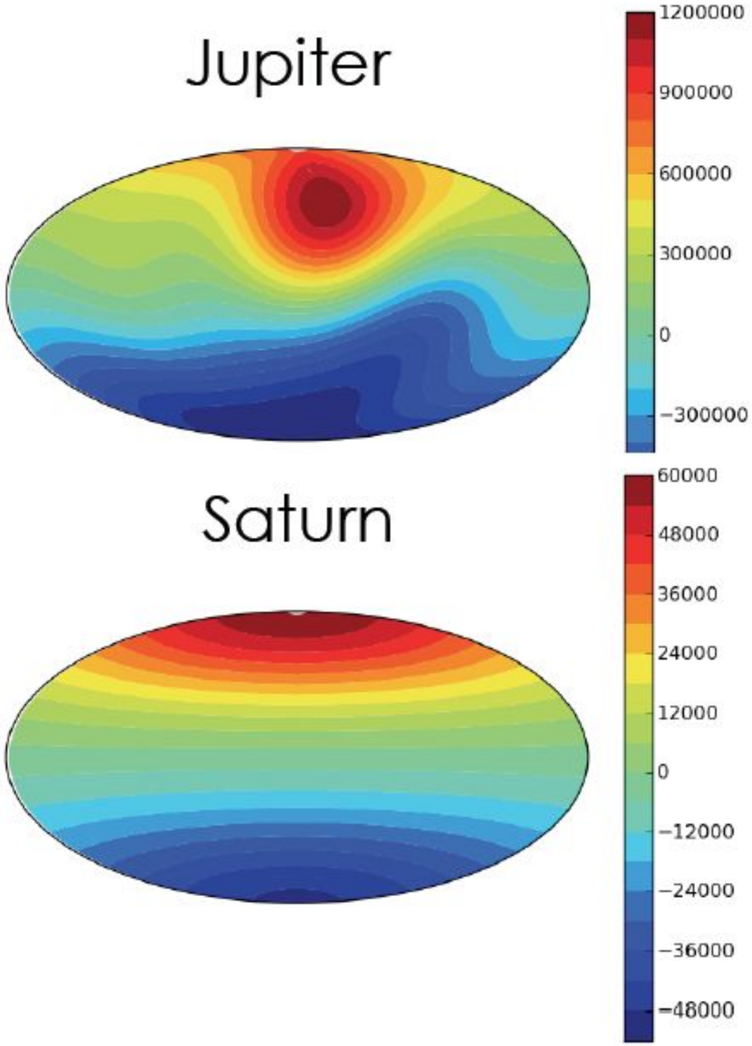
Planet	Dynamo	R_c/R_p	B_s [μT]	Dip. tilt	$\frac{\text{Quadr}}{\text{Dipole}}$
Mercury	Yes (?)	0.75	0.35	$<5^o$?	0.1-0.5
Venus	No	0.55			
Earth	Yes	0.55	44	10.4^o	0.04
Moon	No	0.2 ?			
Mars	No, but in past	0.5			
Jupiter	Yes	0.84	640	9.4^o	0.10
Saturn	Yes	0.6	31	0^o	0.02
Uranus	Yes	0.75	48	59^o	1.3
Neptune	Yes	0.75	47	45^o	2.7
Ganymede	Yes	0.3 ?	1.0	$< 5^o$?	?

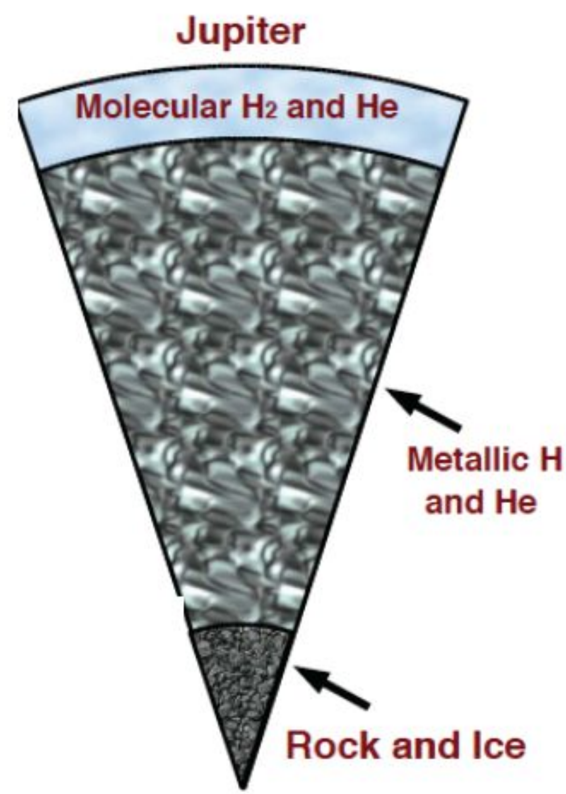
What that means



Christensen

Gas Giants





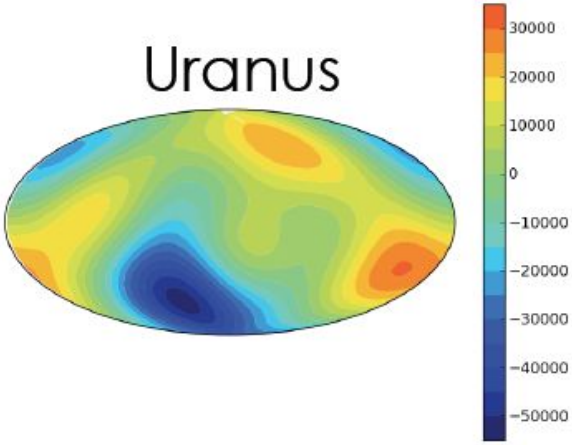
Stanley

- large pressure range → radially variable properties can be important

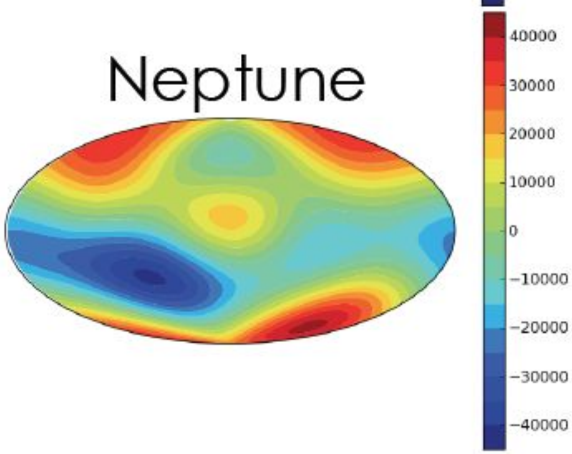
ICE GIANTS

Stanley

Uranus

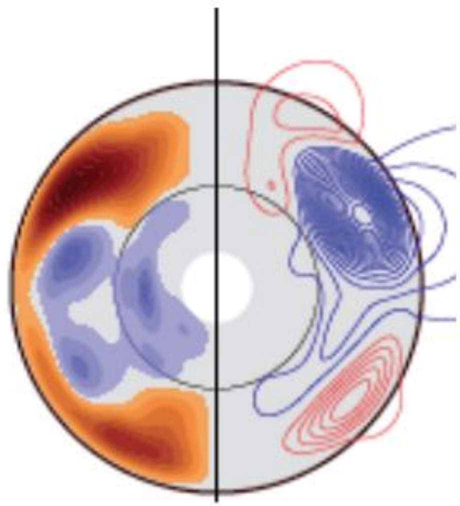


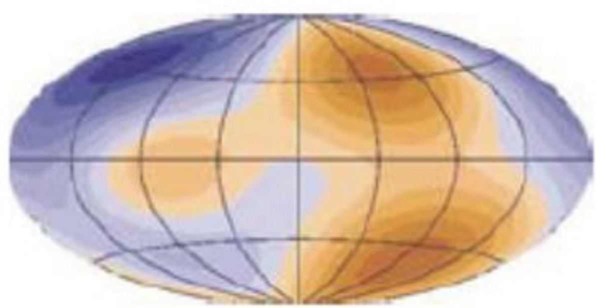
Neptune

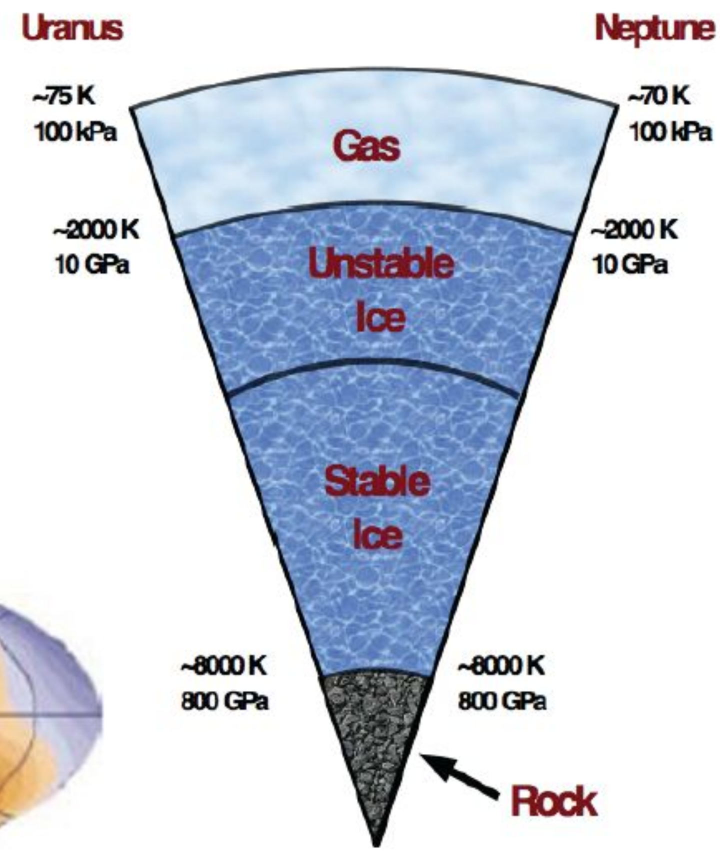


Stanley & Bloxham 2004, 2006

- work in a geometry suggested by low heat flow observations

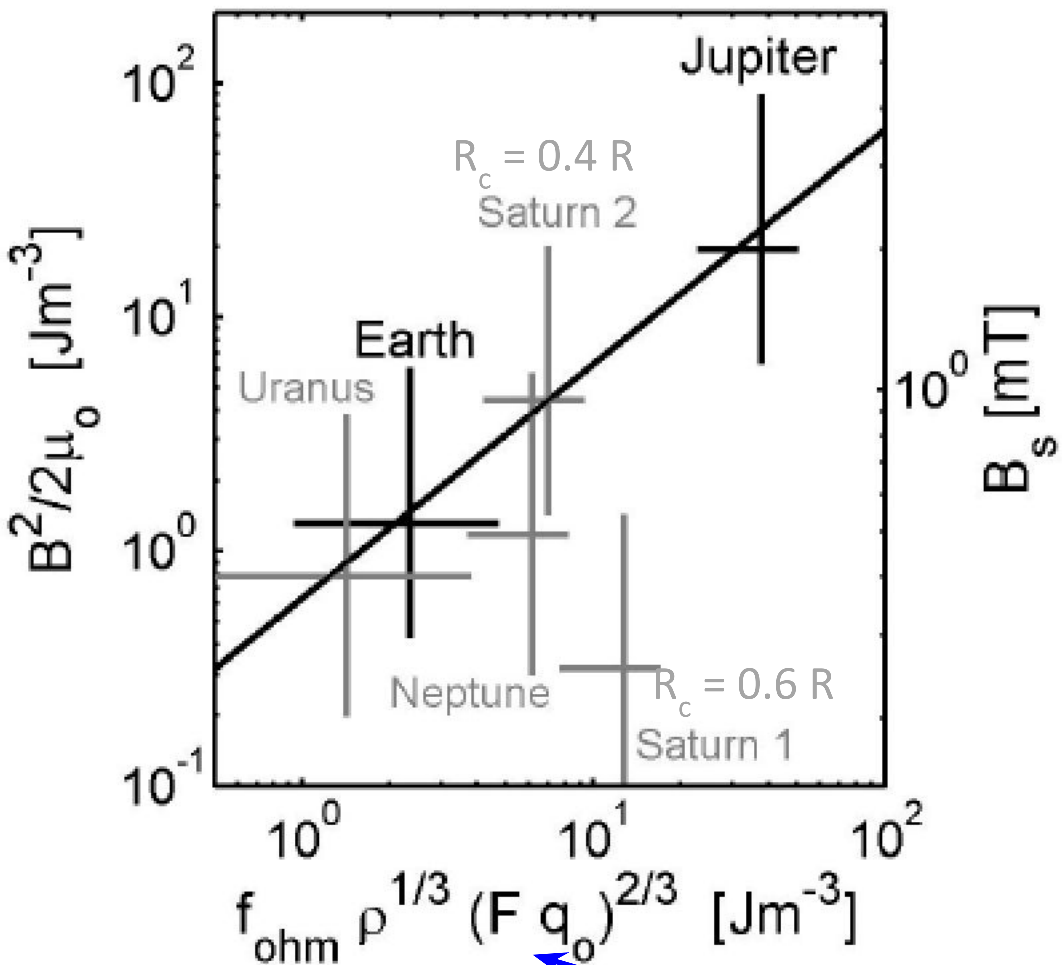






Level of saturation

from Christensen



dimensionless factors

B saturates
(exp growth ends)
when driving power – thermal conduction q_0 – balances Ohmic dissipation

Summary

- Magnetic fields – all from dynamos
  - Conducting fluid
  - Complex motions w/ enough *umph*
- Create complex fields
- Fields evolve in time – reverse occasionally
- Differences from different parameters:

  Rm, Re, Ro