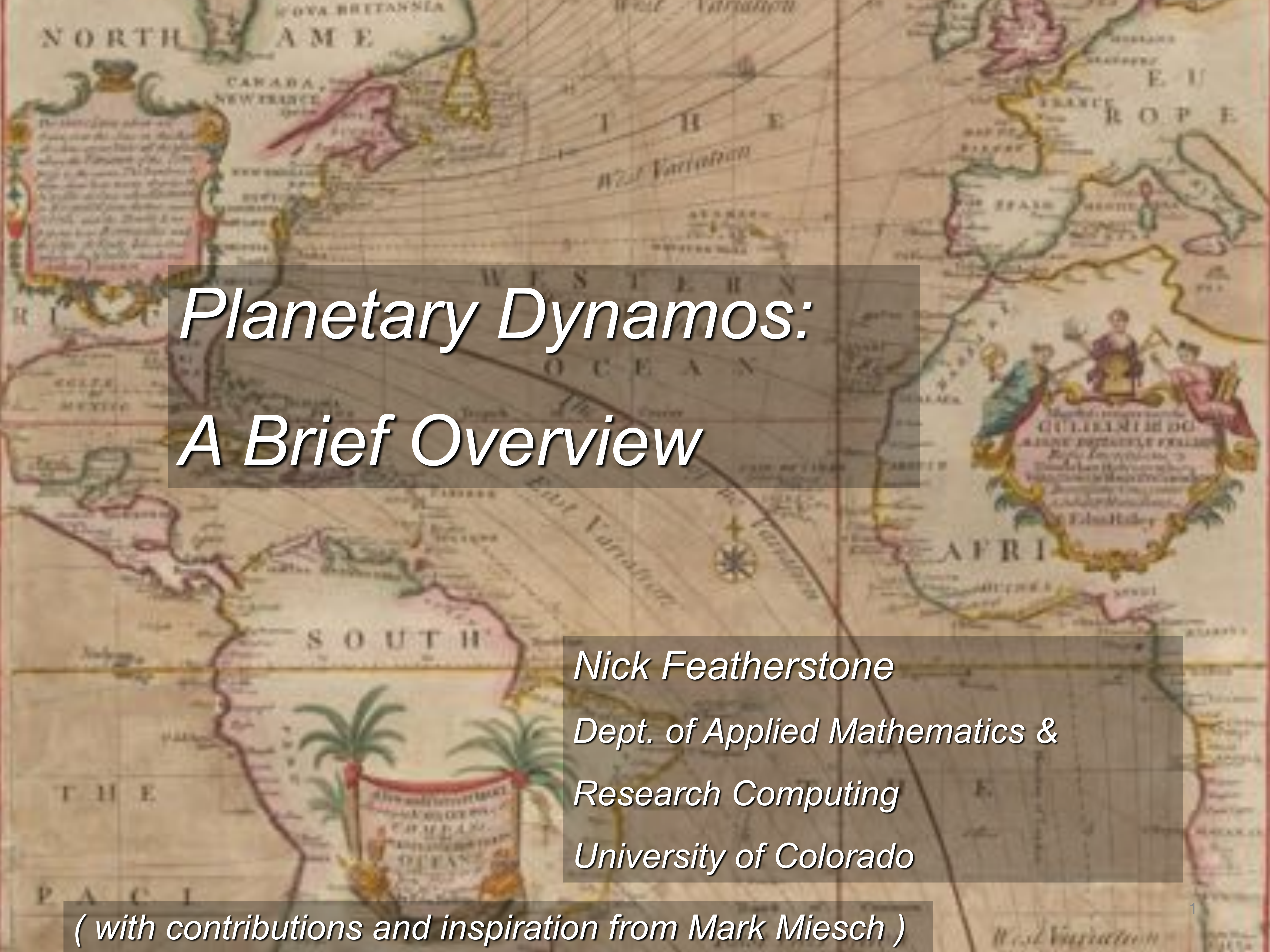


Planetary Dynamos:

A Brief Overview

Nick Featherstone

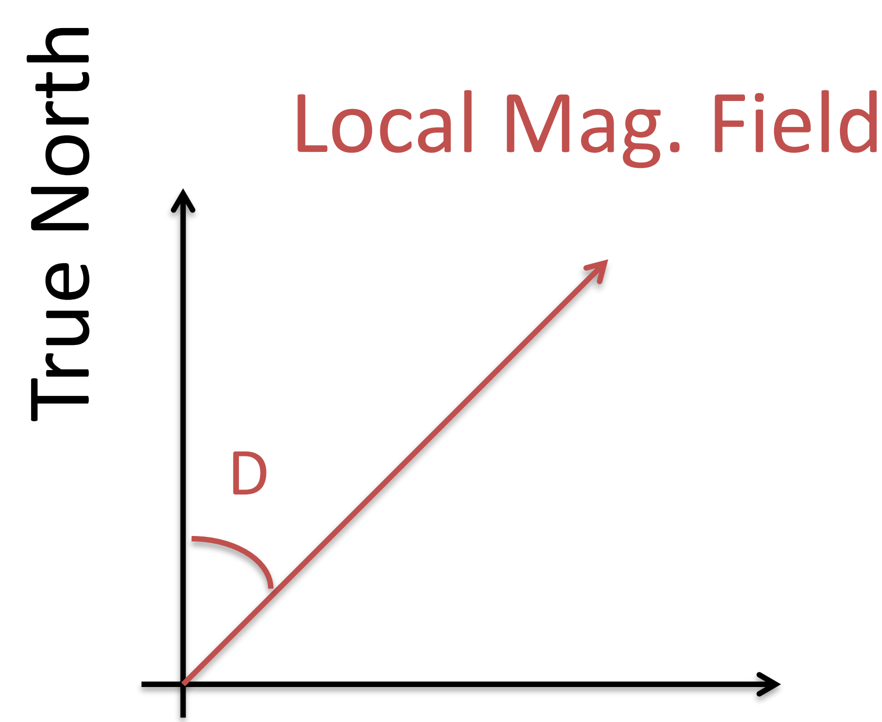
Dept. of Applied Mathematics &

Research Computing

University of Colorado

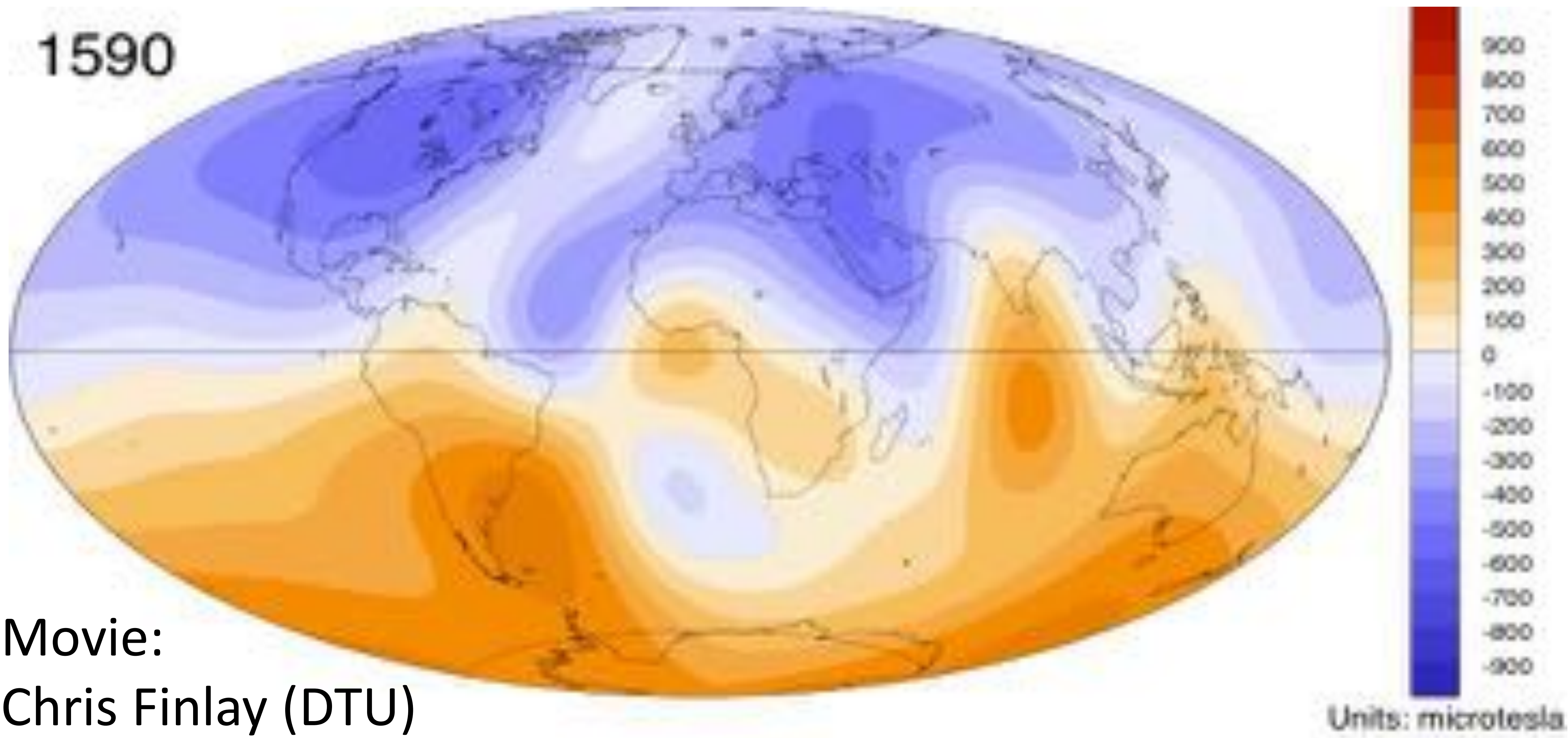
(with contributions and inspiration from Mark Miesch)

Geomagnetic Declination in 1701



[Edmond Haley, 1701]

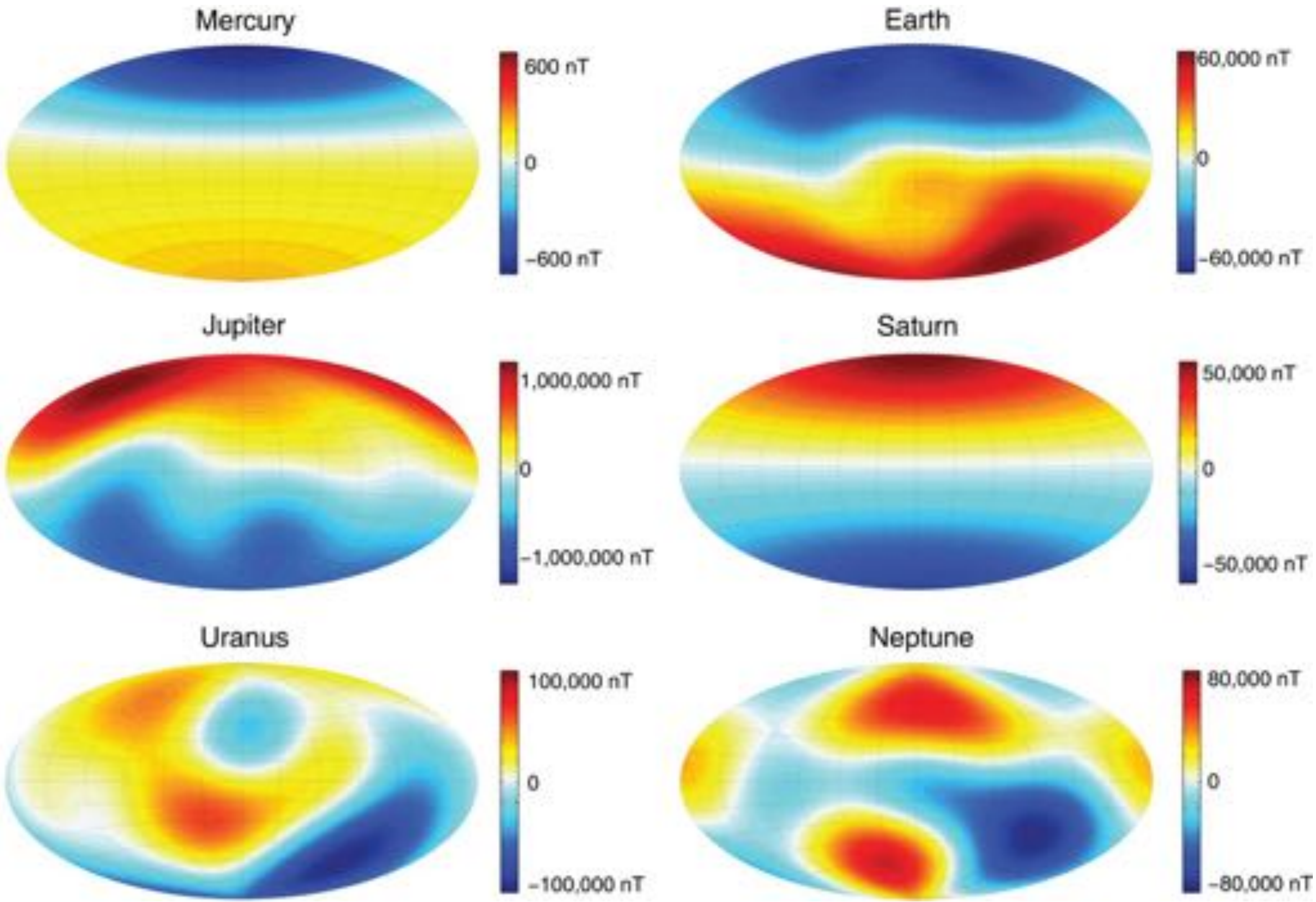
History of Earth's Magnetic Field



Movie:
Chris Finlay (DTU)

Geomagnetism is Dynamic
Something inside the Earth is causing this variation

Most Planets Possess Magnetic Fields



[Cao 2014]

Where are we going?

- Quick review of dynamo fundamentals
- A closer look at rotating convection
- Survey of magnetism in the solar system
- The triumphs and troubles of simulations

MHD Magnetic Induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Comes from Maxwell's equations (Faraday's Law and Ampere's Law)

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}$$ *(Assumes v << c)*

And Ohm's Law

$$\tilde{\mathbf{J}} = \sigma \tilde{\mathbf{E}}$$

electrical conductivity

Magnetic diffusivity

$$\eta = \frac{c^2}{4\pi\sigma}$$

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy

Sink of Magnetic Energy

How would you demonstrate this?

$$E_m = \frac{B^2}{8\pi}$$

(Hint: have a sheet handy with lots of vector identities!)

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy

Sink of Magnetic Energy

$$\frac{\partial E_m}{\partial t} = -\nabla \cdot \mathbf{F}_P - \frac{\mathbf{v}}{c} \cdot (\mathbf{J} \times \mathbf{B}) - \Phi_o$$

Poynting Flux

$$\mathbf{F}_P = \mathbf{E} \times \mathbf{B} = \left[\frac{\eta}{c} \mathbf{J} - \frac{1}{4\pi} (\mathbf{v} \times \mathbf{B}) \right] \times \mathbf{B}$$

Ohmic Heating

$$\Phi_o = \frac{4\pi\eta}{c^2} J^2$$

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy ~ U B / D

Sink of Magnetic Energy ~ η B / D^2

$$\mathrm{Rm} = \frac{UD}{\eta}$$

If Rm >> 1 the source term is much bigger than the sink term

....Or is it???

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy
~ U B / D

Sink of Magnetic Energy
~ η B / δ^2

δ can get so small that the two terms are comparable

It's not obvious which term will "win" - it depends on the subtleties of the flow, including geometry & boundary conditions

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy
$\sim U\,B\,/\,D$

Sink of Magnetic Energy
$\sim \eta\,B\,/\,\delta^2$

What is a Dynamo? (A corollary)

A dynamo must sustain the magnetic energy (through the conversion of kinetic energy) against Ohmic dissipation

The need for a Dynamo

If v = 0 and η = constant then the induction equation becomes

$$\frac{\partial \mathbf{B}}{\partial t} = -\eta \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{B} = \eta \nabla^2 \mathbf{B}$$

The field will diffuse away (dissipation of magnetic energy) on a time scale of

$$\tau_d \approx \frac{D^2}{\eta}$$

A more careful calculation for a planet gives

$$\tau_d \approx \frac{R^2}{\pi^2 \eta}$$

Earth: τ_d ~ 80,000 yrs

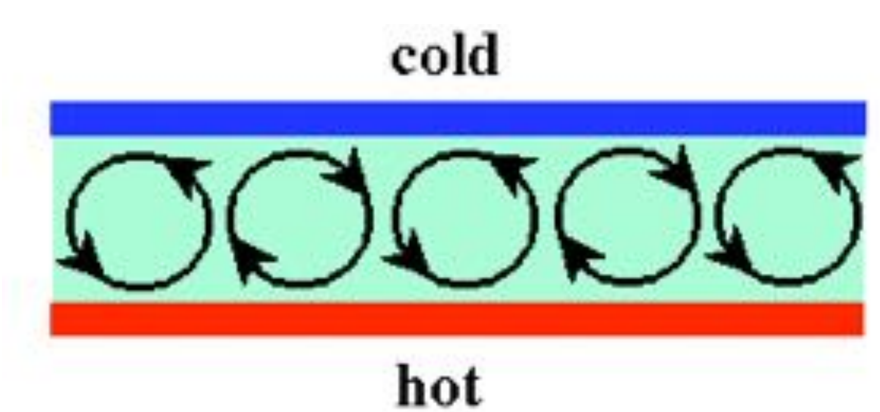
Jupiter: τ_d ~ 30 million yrs

Planetary fields must be maintained by a dynamo or they would have decayed by now!

Conditions for a Planetary (or Stellar) Dynamo

Absolutely necessary

- **An electrically conducting fluid**
  - Stars: plasma
  - Terrestrial planets: molten metal (mostly iron)
  - Jovian planets: metallic hydrogen (maybe molecular H)
  - Ice Giants: water/methane/ammonia mixture
  - Icy moons: salty water
- **Fluid motions**
  - Usually generated by buoyancy (convection)
- **Rm >> 1**
  - Too much ohmic diffusion will kill a dynamo



Conditions for a Planetary (or Stellar) Dynamo

Not strictly necessary but it (usually) helps

- **Rotation**
  - **Good**: helps to build strong, large-scale fields (promotes magnetic self-organization)
  - **Bad**: can suppress convection (though this is usually not a problem for planets)

- **Turbulence** (low viscosity / Re >> 1)
  - **Good**: Chaotic fluid trajectories good at amplifying magnetic fields (chaotic stretching)
  - **Bad**: can increase ohmic dissipation

The MHD Induction Equation: Alternate View

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\boldsymbol{B}\nabla\cdot\boldsymbol{v} + \boldsymbol{B}\cdot\nabla\boldsymbol{v} - \boldsymbol{v}\cdot\nabla\boldsymbol{B} - \nabla\times(\eta\boldsymbol{\nabla}\times\boldsymbol{B})$$

compression

shear production

advection

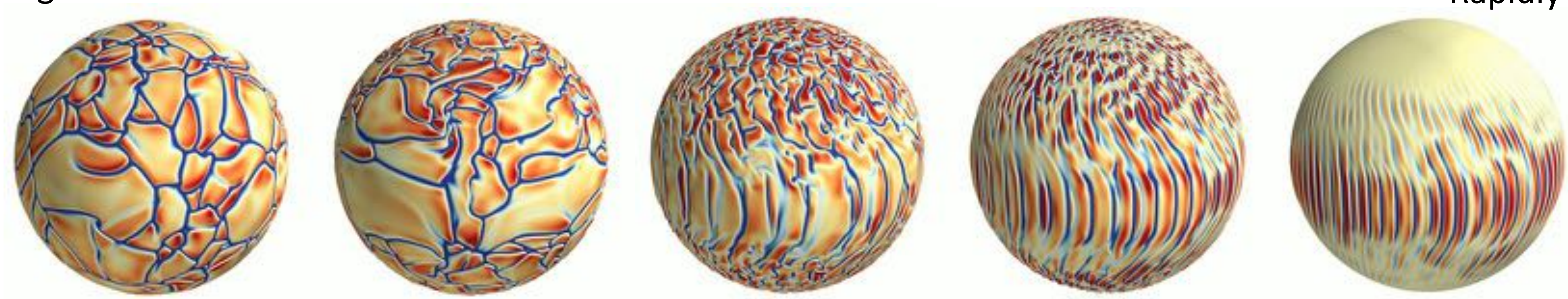
diffusion

shear production ← Rotation → Large-Scale Shear (differential rotation)

Rotation → Small-Scale Shear (helical rolls)

Convection Simulations

Non-rotating → Rapidly-rotating



red upflow

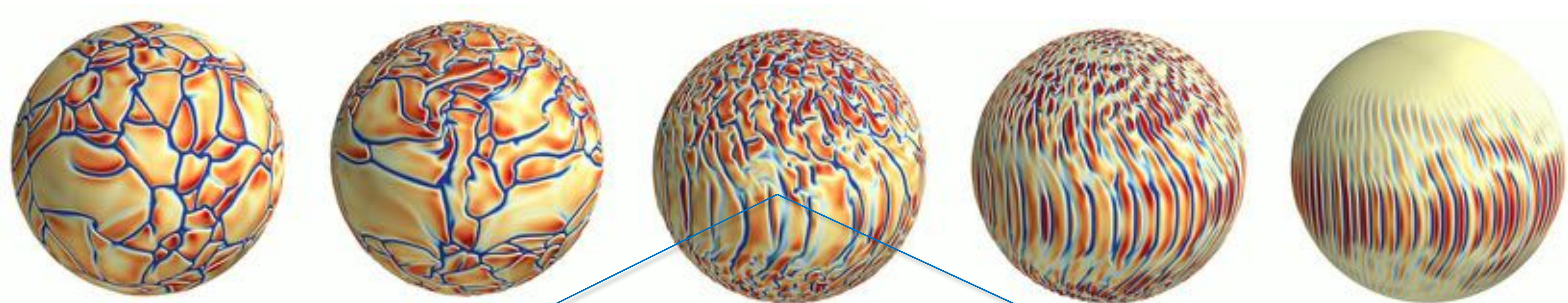
blue downflow

Rotating convection naturally generates both small-scale and large-scale shear!

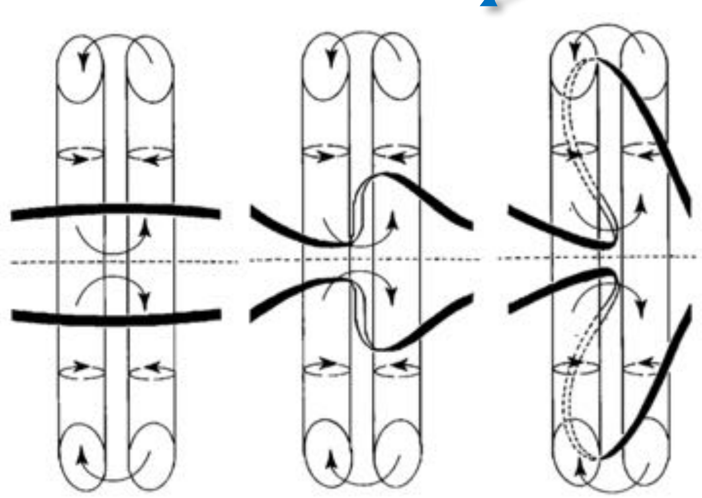
Rotation Yields Helical Convection

Non-rotating
or
Fast Convection

Rapidly-rotating
or
Slow Convection

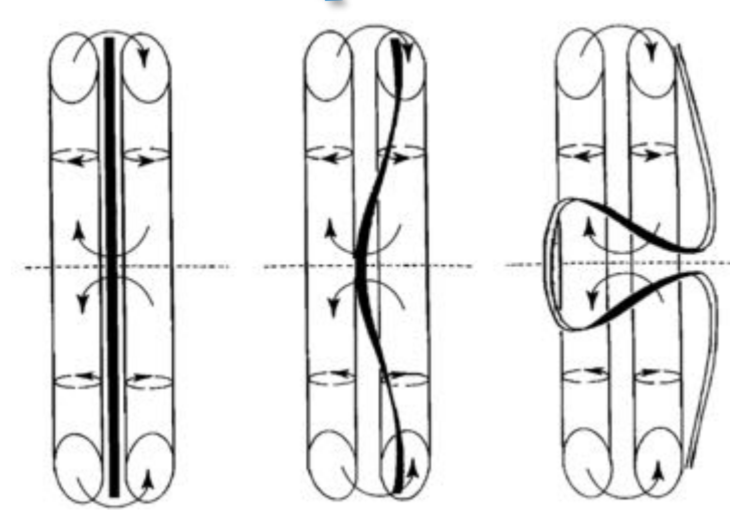


"alpha-effect"



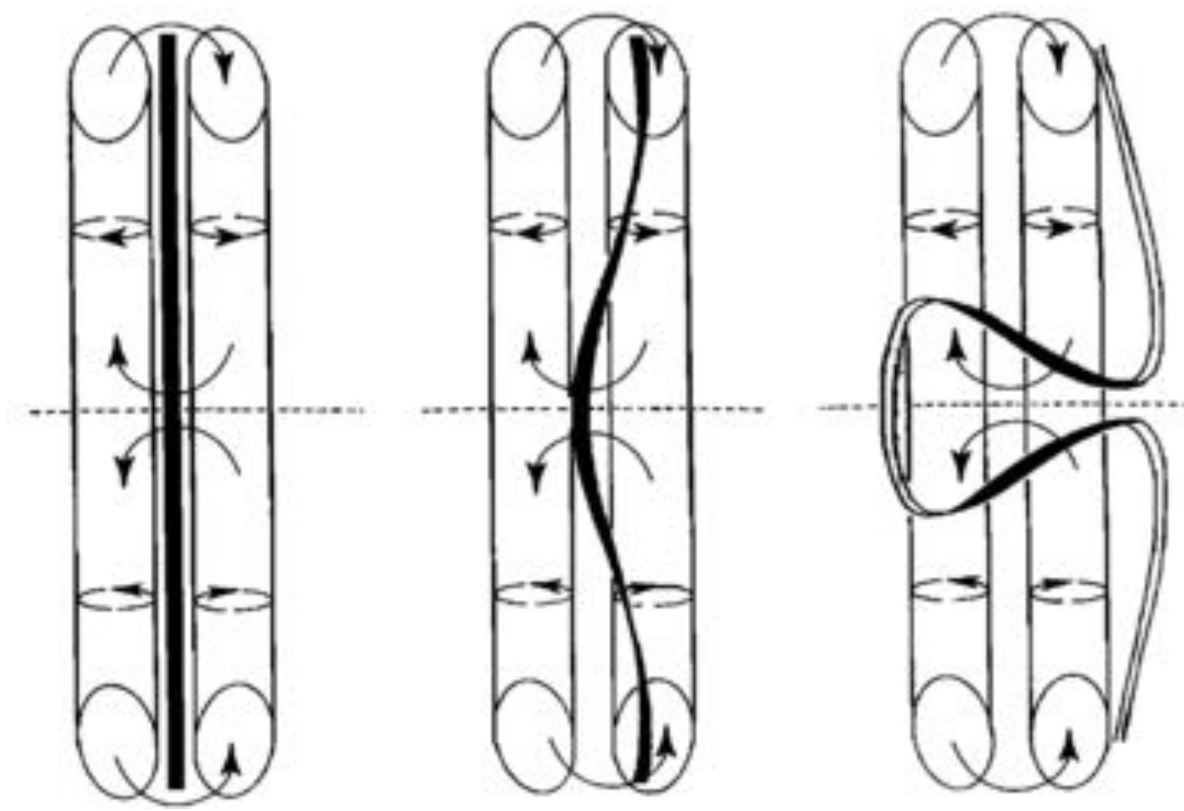
toroidal field → poloidal field

Olson et al., 1999, JGR



poloidal field → toroidal field

Convection…
… Rotation?



Helical rolls (or their turbulent counterparts) probably form the central engine of most planetary and stellar dynamos.

So where do they come from?

The (hydro) momentum equation

- Consider incompressible flow with constant diffusivities

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} = -\rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} - 2\rho(\boldsymbol{\Omega} \times \boldsymbol{u}) + \rho \boldsymbol{g} - \boldsymbol{\nabla} P + \rho \upsilon \nabla^2 \boldsymbol{u}$$

- Consider perturbations about background state:

$$\rho = \rho' + \bar{\rho} \qquad \rho' \ll \bar{\rho} \qquad \bar{\rho} = constant$$

$$P = P' + \overline{P(r)} \qquad P' \ll \bar{P}$$

The (hydro) momentum equation

- Subtract out hydrostatic balance
- Divide by $\bar{\rho}$

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\boldsymbol{u}\cdot\nabla\boldsymbol{u} - 2(\boldsymbol{\Omega}\times\boldsymbol{u}) + \frac{\rho'}{\bar{\rho}}\boldsymbol{g} - \frac{1}{\bar{\rho}}\boldsymbol{\nabla}P' + \upsilon\nabla^2\boldsymbol{u}$$

- Recast density perturbation in terms of temperature

$$\frac{\rho'}{\bar{\rho}} = -\alpha T'$$

Hotter than surroundings = low density

Cooler than surroundings = high density

$$\alpha > 0$$

The (hydro) momentum equation

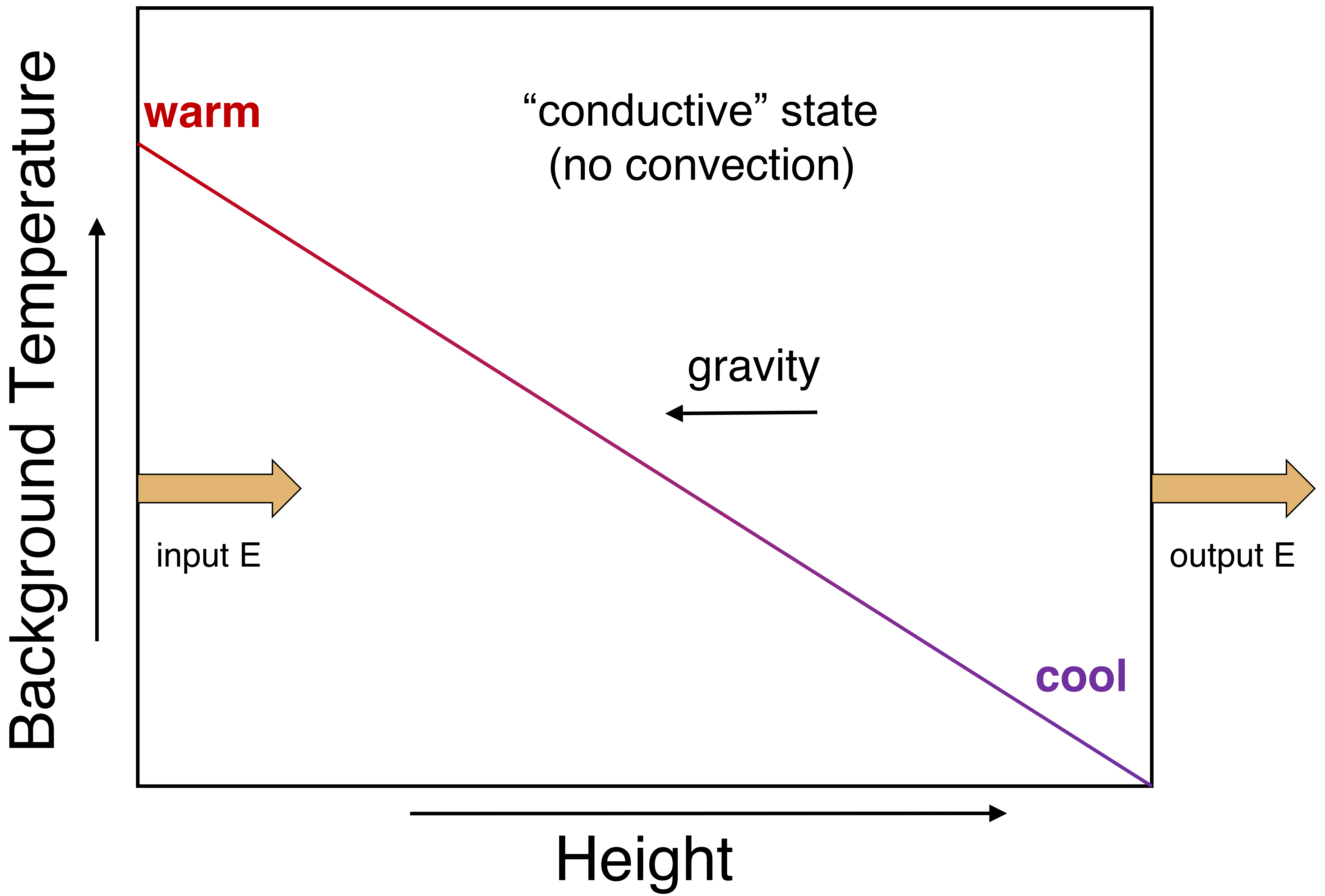
- The end result

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\boldsymbol{u} \cdot \nabla \boldsymbol{u} - 2(\boldsymbol{\Omega} \times \boldsymbol{u}) + \alpha T' g \hat{\boldsymbol{r}} - \frac{1}{\bar{\rho}} \boldsymbol{\nabla} P + \upsilon \nabla^2 \boldsymbol{u}$$

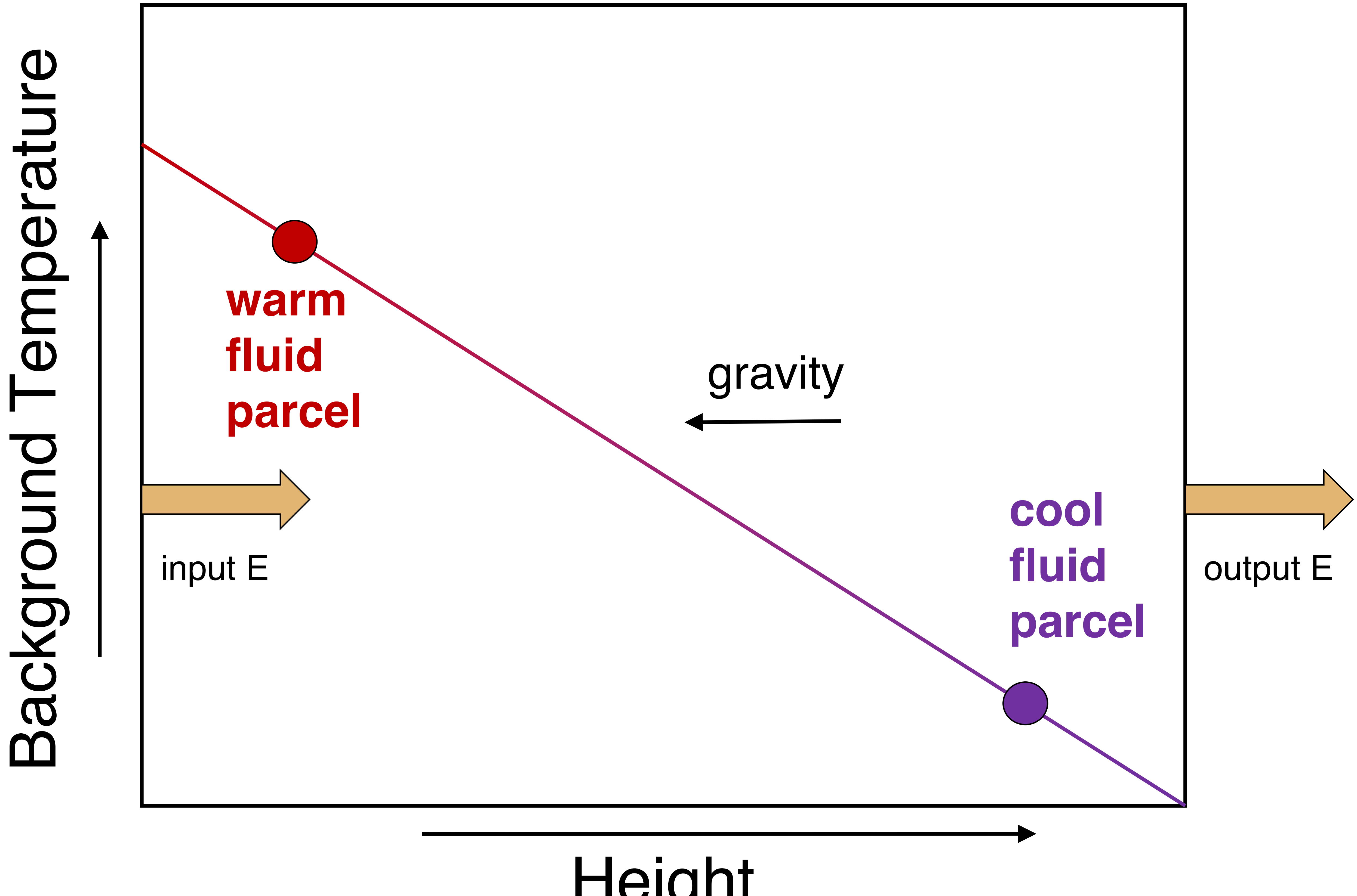
$$\frac{\rho'}{\bar{\rho}} = -\alpha T' \qquad \alpha > 0$$

- Hot fluid rises
- Cool fluid sinks
- This leads to convection (under the proper circumstances)

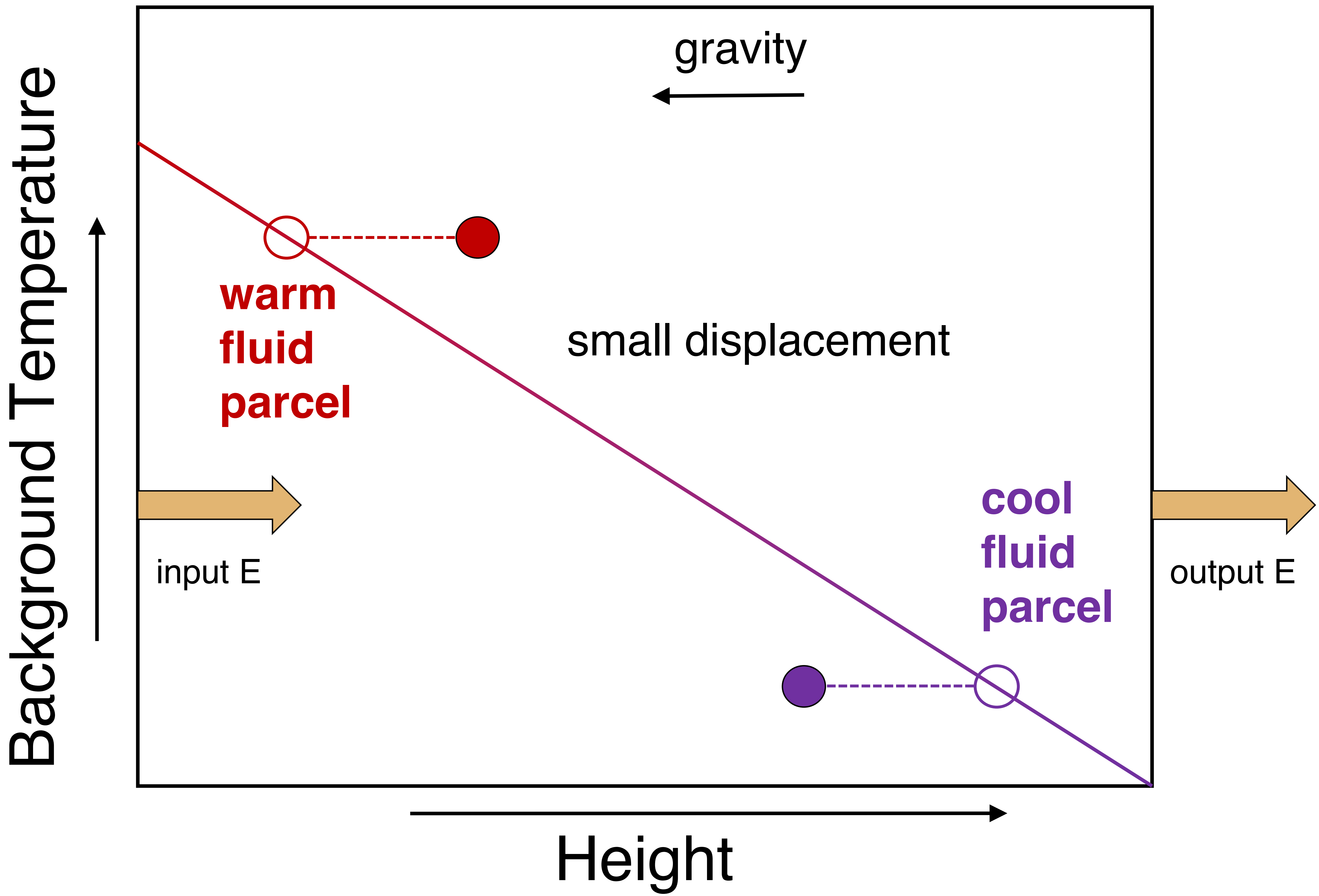
Convection 101



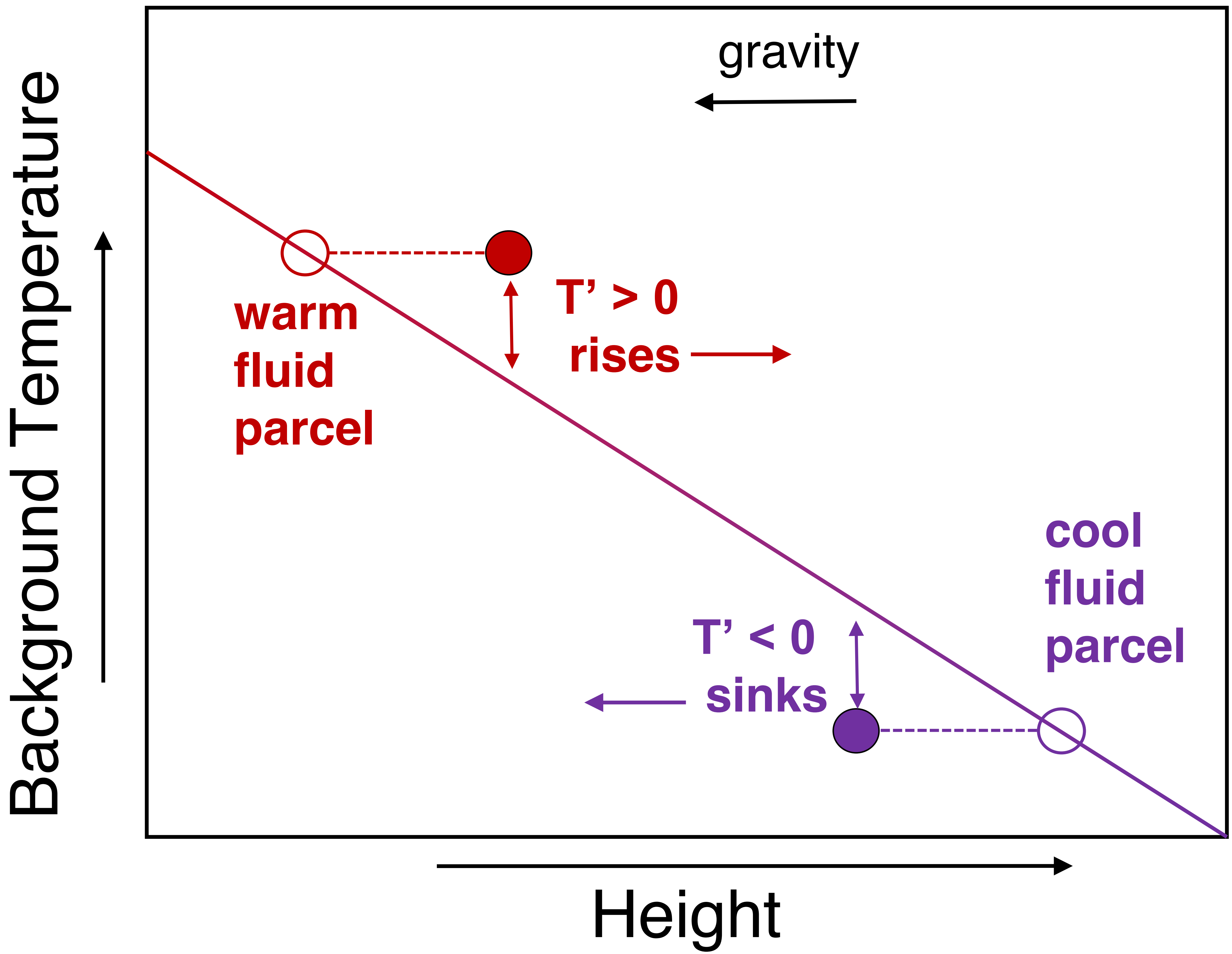
Convection 101



Convection 101

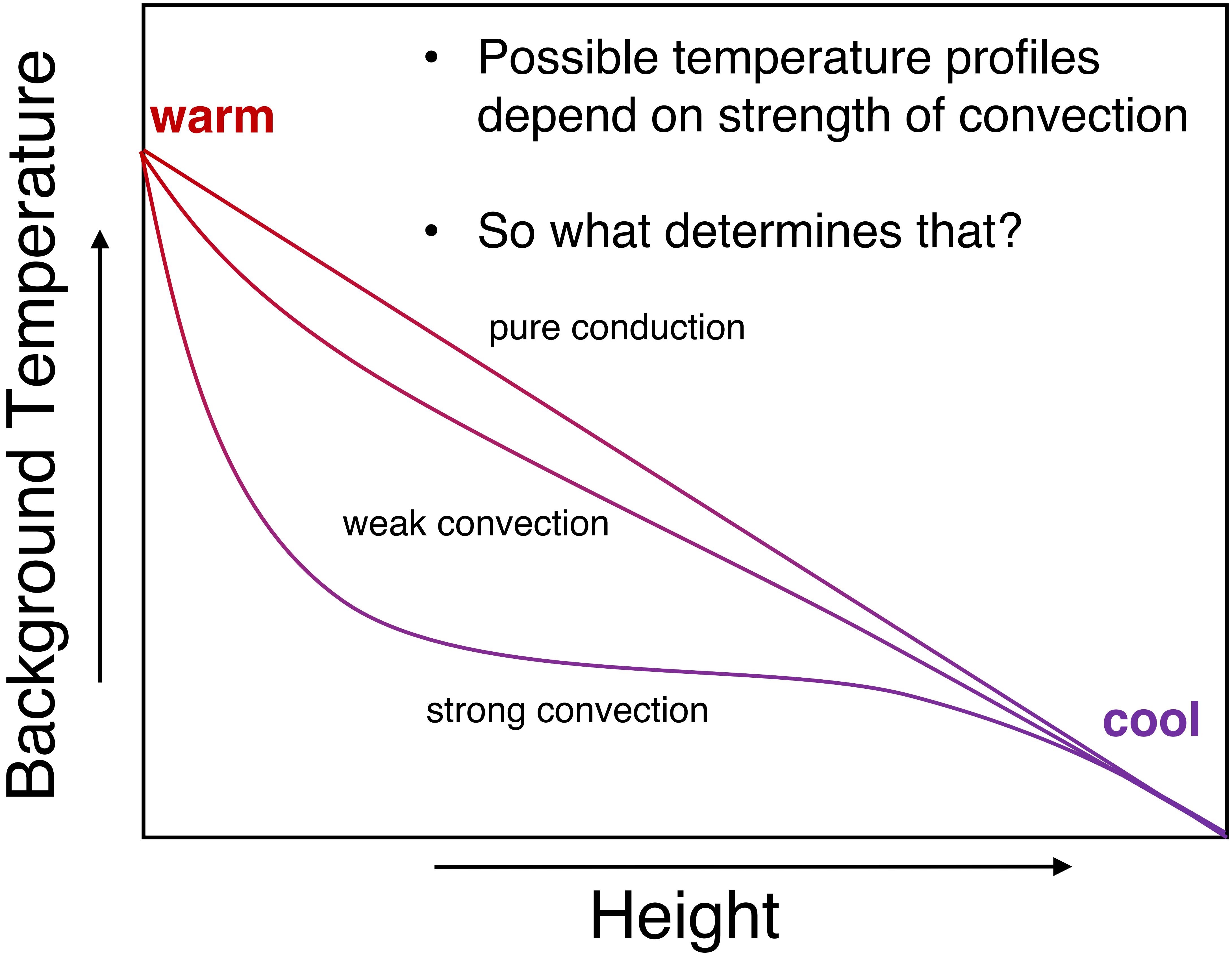


Convection 101



Convection 101

- Possible temperature profiles depend on strength of convection
- So what determines that?



The Internal Energy Equation

- Consider incompressible flow with constant diffusivities

diffusion

$$\frac{\partial T}{\partial t} = -\boldsymbol{u} \cdot \nabla T + \kappa \nabla^2 T$$

advection

- Competition between advection and diffusion

The competition: buoyancy vs. diffusion

- As a fluid parcel rises or falls, it also diffuses
- If diffusion is too large, it dissipates heat/momentum before making it very far
- We can quantify this

Exercise: The diffusive timescale

- Consider the 1-D diffusion equation:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

- Seek a solution of the form:

$$T = A e^{\frac{t}{\tau}} \sin(\frac{\pi}{L} x)$$

- What is τ ? Is it positive or negative?

Exercise: The diffusive timescale

- Solution:

$$\tau = \frac{L^2}{\kappa \pi^2}$$

- Neglect factor of π^2:

$$\tau \sim \frac{L^2}{\kappa}$$

- Diffusion time for length scale L

Important Timescales

thermal diffusion time

$$\tau_\kappa \sim \frac{L^2}{\kappa}$$

viscous diffusion time

$$\tau_\nu \sim \frac{L^2}{\nu}$$

buoyancy timescale?

$$\tau_B \sim ?$$

Exercise: buoyancy timescale

- Consider simplified momentum equation:

$$\frac{\partial u}{\partial t} = \alpha \tilde{T} g$$

- What is freefall time (τ_B) over a distance L? (assume $\tilde{T}$ is constant)

$$\tau_B = \sqrt{\frac{2L}{\alpha \tilde{T} g}}$$

Important Timescales

thermal

$$\tau_\kappa \sim \frac{L^2}{\kappa}$$

viscous

$$\tau_\nu \sim \frac{L^2}{\nu}$$

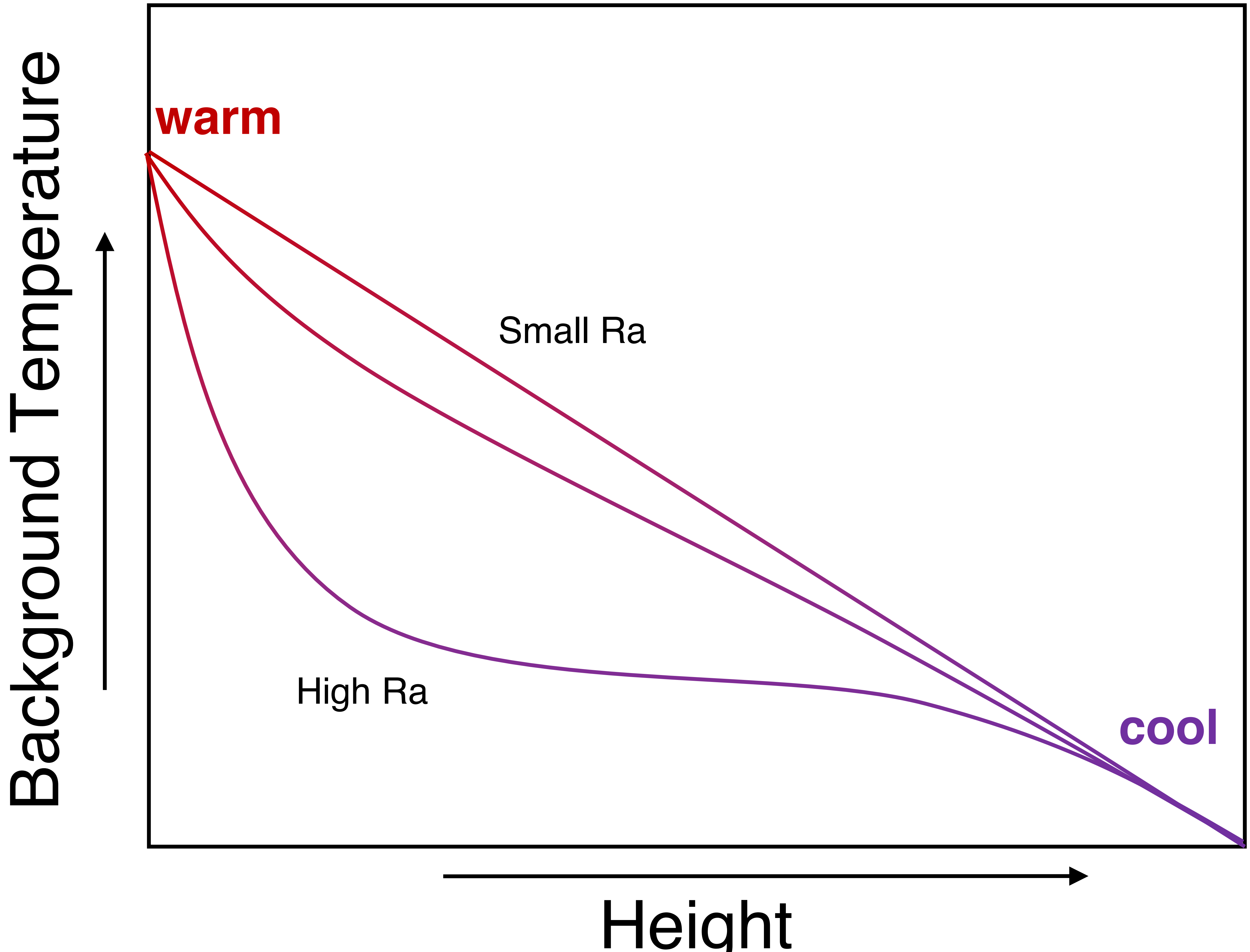
buoyancy

$$\tau_B \sim \sqrt{\frac{L}{\alpha \tilde{T} g}}$$

Can quantify competition between buoyantly driven advection and diffusion via the Rayleigh number Ra:

$$Ra = \left(\frac{\tau_\kappa}{\tau_B}\right)\left(\frac{\tau_\upsilon}{\tau_B}\right) = \frac{\alpha \tilde{T} g L^3}{\upsilon \kappa}$$

Convection 101



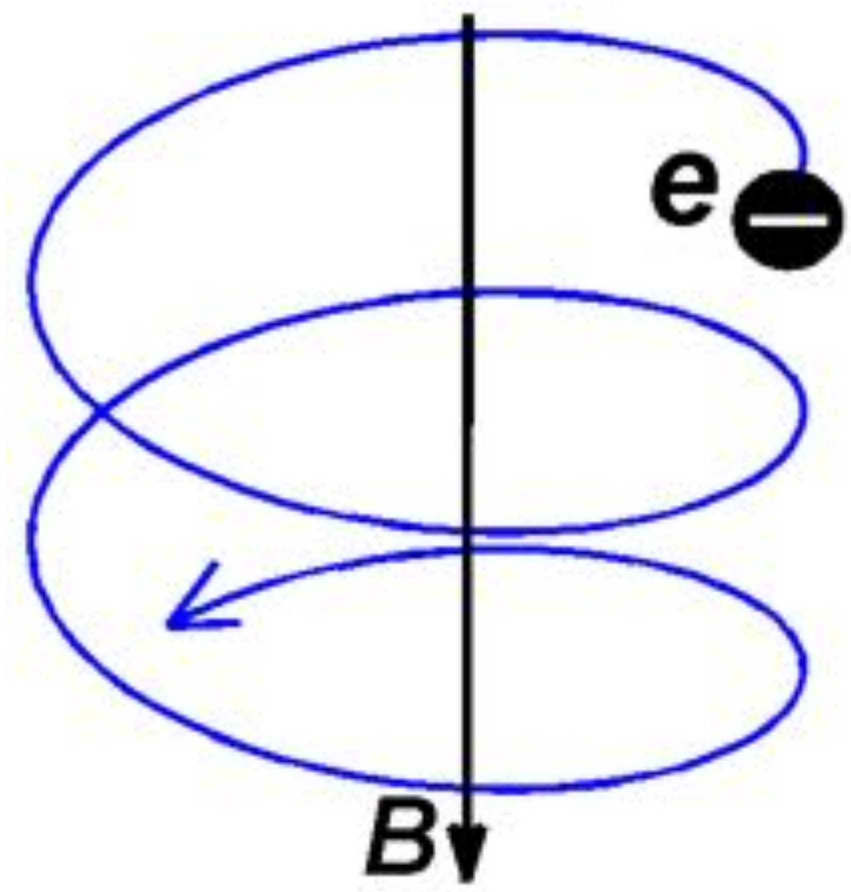
Why Helical Rolls?

Coriolis Force:

$$\frac{\partial \boldsymbol{u}}{\partial t} = 2\boldsymbol{u} \times \boldsymbol{\Omega}$$

Lorentz Force:

$$\frac{\partial \boldsymbol{u}}{\partial t} = q\boldsymbol{u} \times \boldsymbol{B}$$



Some Important Numbers

$$Ra = \frac{\alpha \tilde{T} g L^3}{\upsilon \kappa}$$

$$\frac{\text{Dissipation Timescale}}{\text{Buoyancy Timescale}}$$

$$\mathrm{Ro} = \frac{U}{2\Omega D}$$

$$\frac{\text{Rotational Timescale}}{\text{Convective Timescale}}$$

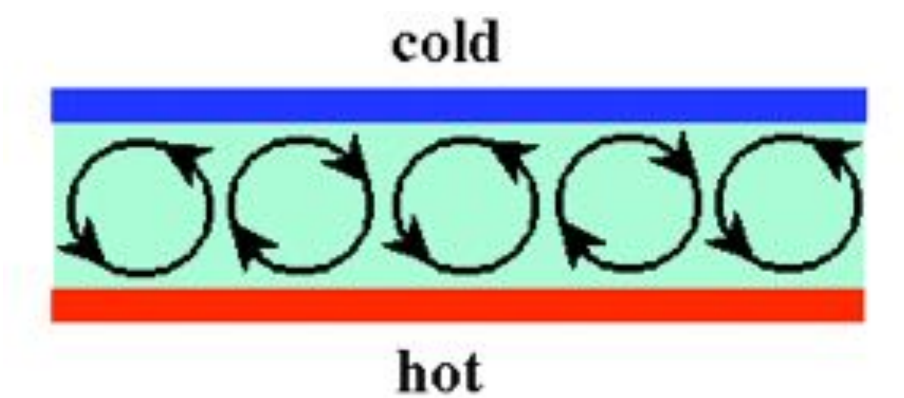
$$\mathrm{Ek} = \frac{\nu}{2\Omega D^2}$$

$$\frac{\text{Rotational Timescale}}{\text{Viscous Timescale}}$$

Understanding the Dynamics

Conservation of momentum in MHD

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\left(\rho \mathbf{v} \cdot \nabla\right) \mathbf{v} - 2\rho \left(\mathbf{\Omega} \times \mathbf{v}\right) - \nabla P + \rho \mathbf{g} + c^{-1} \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathcal{D}$$



Convection established by buoyancy

But rotation exerts an overwhelming influence
Coriolis accelerations happen quickly (days) compared to convection and dynamo time scales (hundreds to thousands of years)

$$\mathrm{Ro} = \frac{U}{2\Omega D} << 1 \qquad \mathrm{Ek} = \frac{\nu}{2\Omega D^2} << 1$$

Dynamical Balances

$$c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\mathbf{\Omega}\times\mathbf{v}\right) + \mathbf{\nabla}P - \rho\mathbf{g}$$

Now set B = 0 and assume that $\nabla\rho$ is mainly radial

Then the ϕ component of the curl gives (anelastic approximation):

$$\mathbf{\Omega}\cdot\mathbf{\nabla}\left(\rho\mathbf{v}\right) = \frac{\partial}{\partial z}\left(\rho\mathbf{v}\right) = 0$$

Taylor-Proudman Theorem

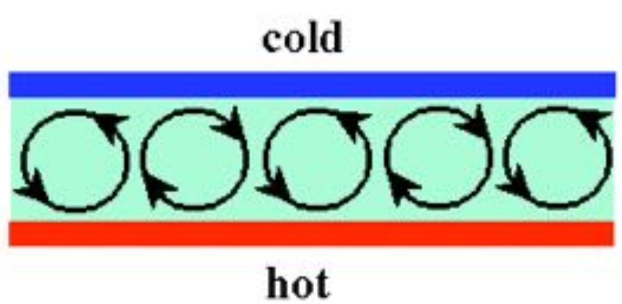
Incompressible version:

$$\frac{\partial \mathbf{v}}{\partial z} = 0$$

Rapidly rotating flows tend to align with the rotation axis

Question

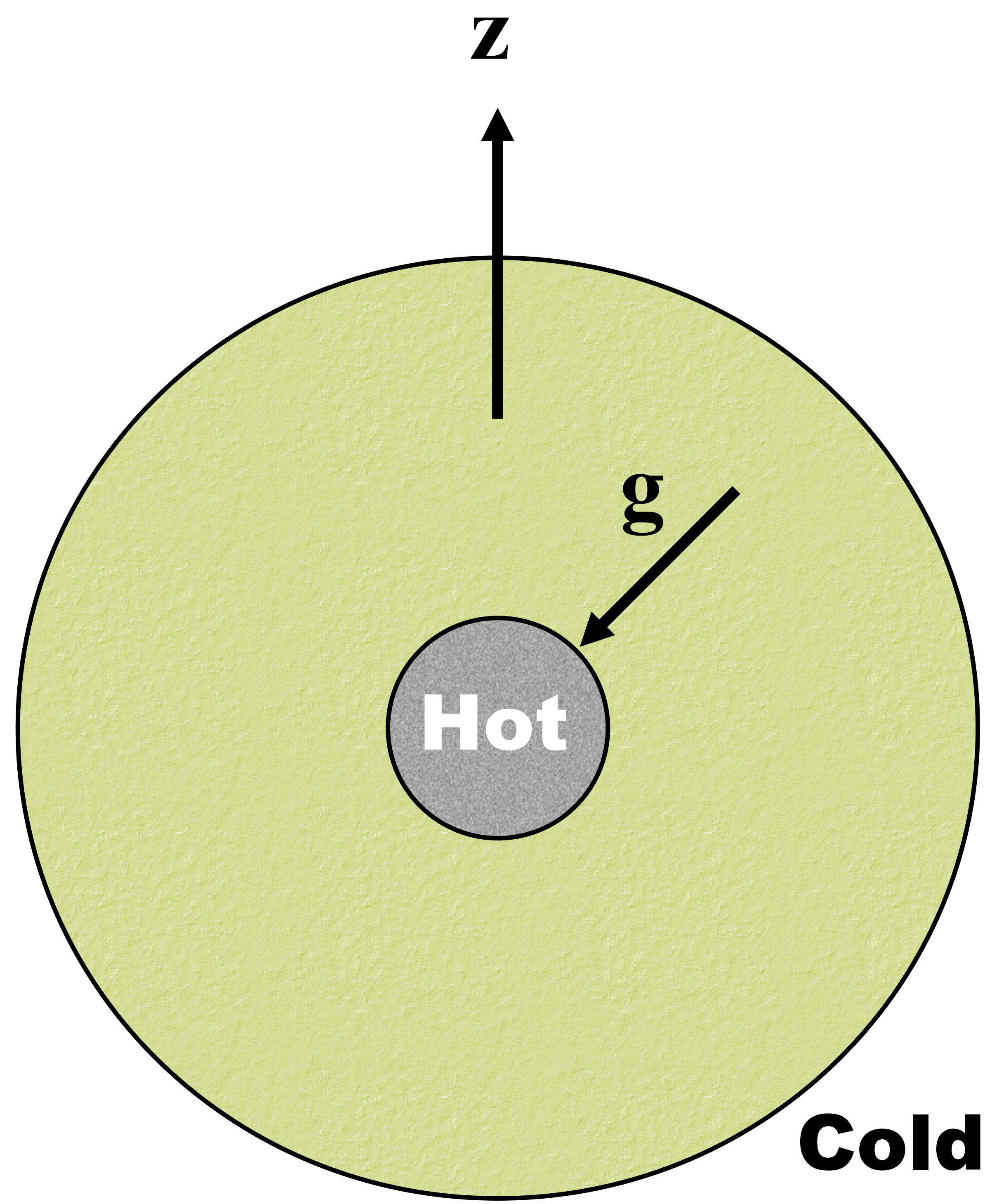
What might convection look like in a rapidly-rotating spherical shell



How can you get the heat out while still satisfying the Taylor-Proudman theorem

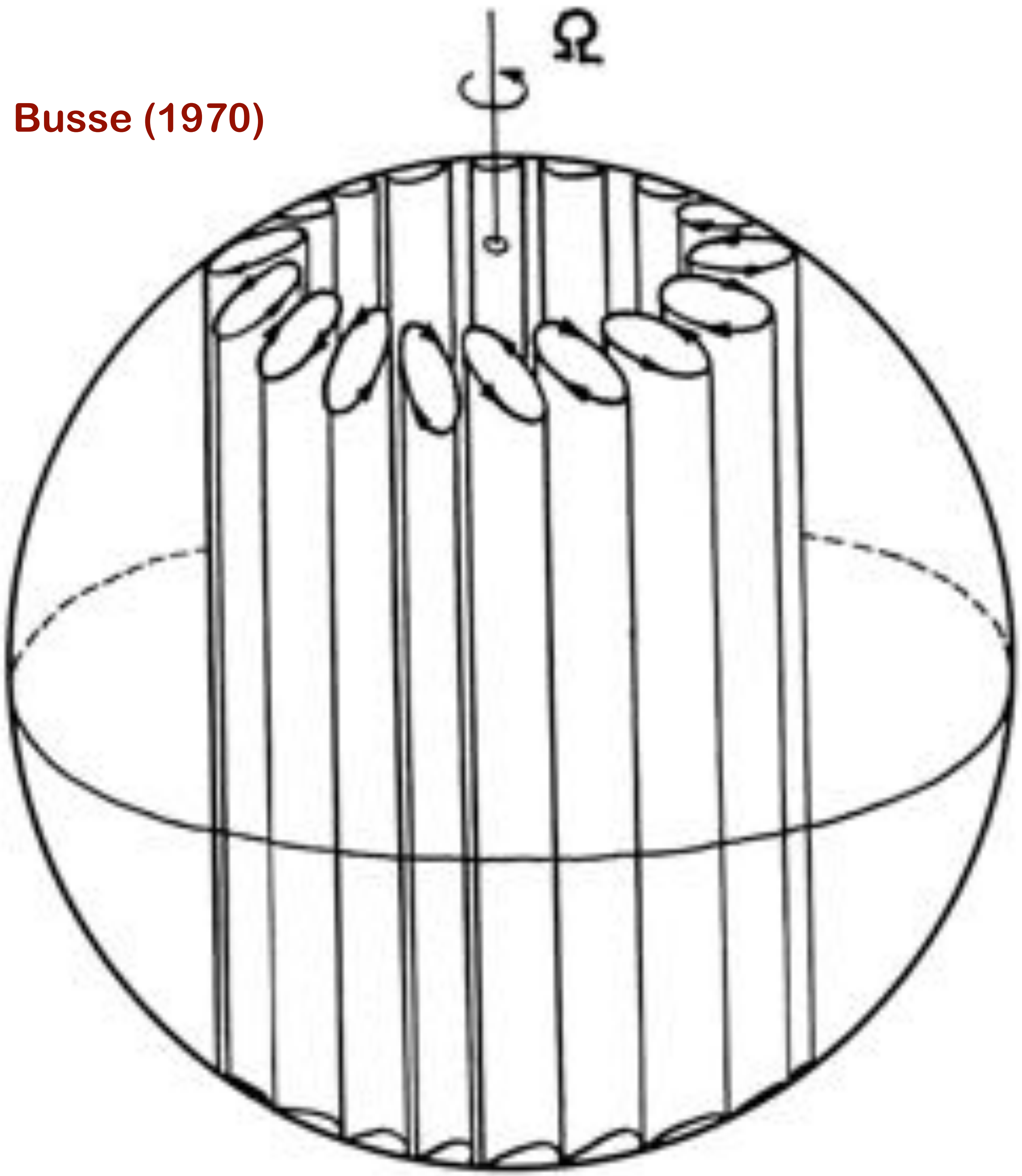
$$\frac{\partial \mathbf{v}}{\partial z} = 0$$

Can you satisfy it everywhere?



Linear Theory

Busse (1970)



The most unstable convective modes in a rapidly-rotating, weakly-stratified shell are

Busse columns

aka

Banana Cells

The preferred longitudinal wavenumber (m) scales as

$$Ek^{-1/3}$$

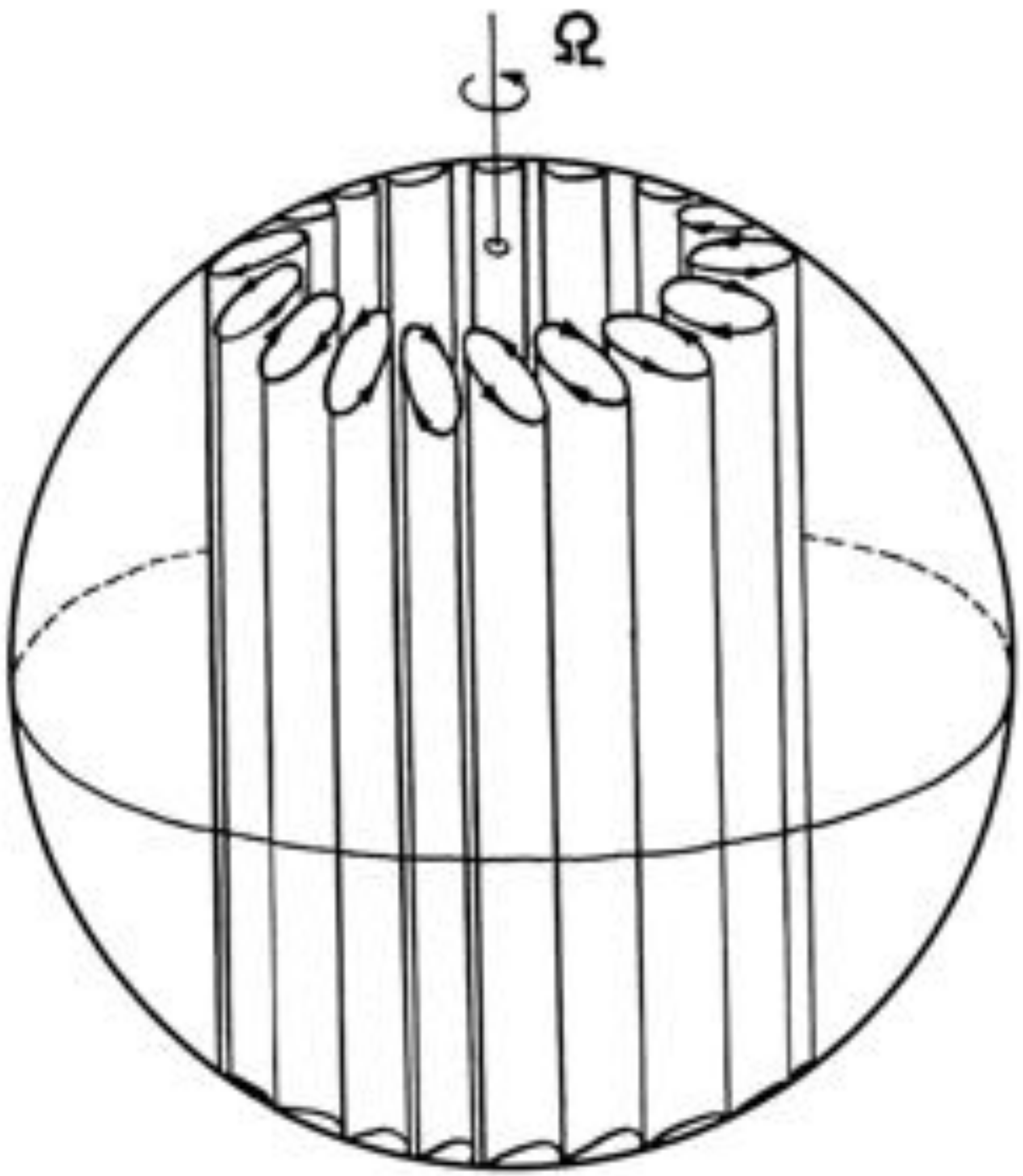
Coriolis vs viscous diffusion

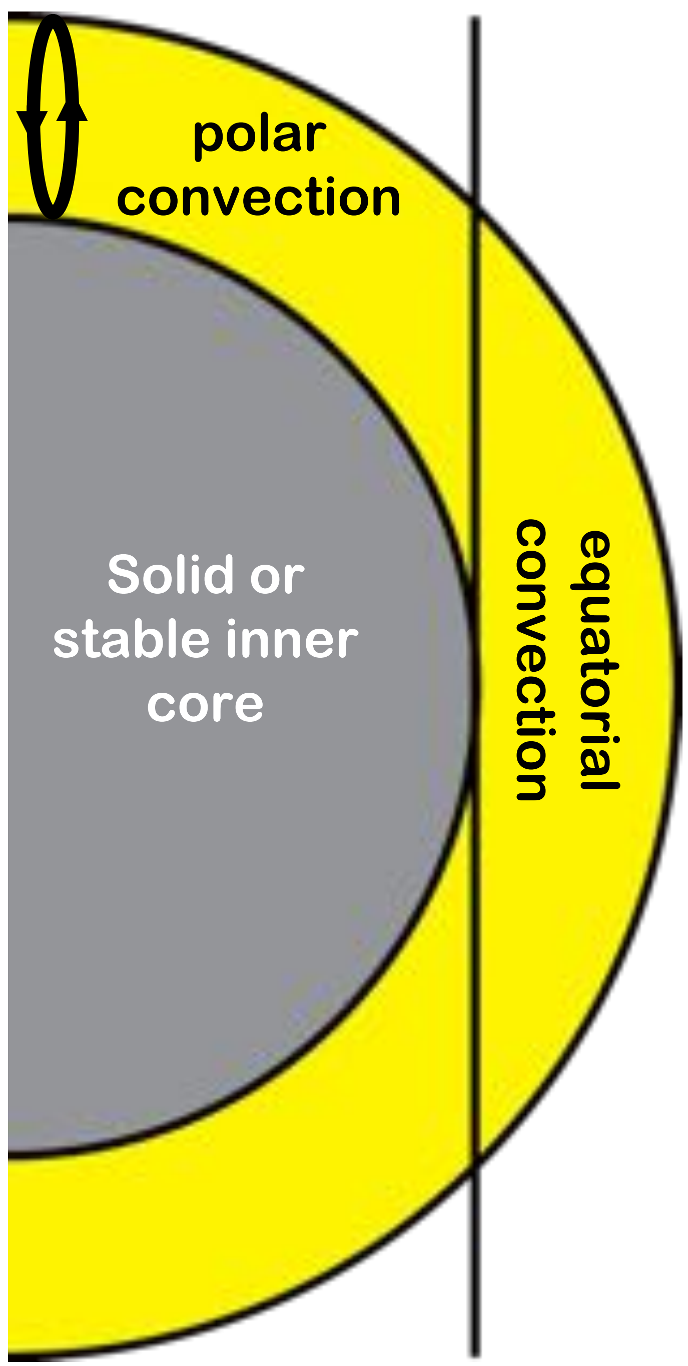
Linear Theory

The

Tangent Cylinder

Delineates two distinct dynamical regimes





Implication of the Taylor-Proudman theorem

Earth

Dynamo!

Field strength
~ 0.4 G

Dipolarity
~ 0.61

Tilt
~ 10°

Archetype of a terrestrial planet!

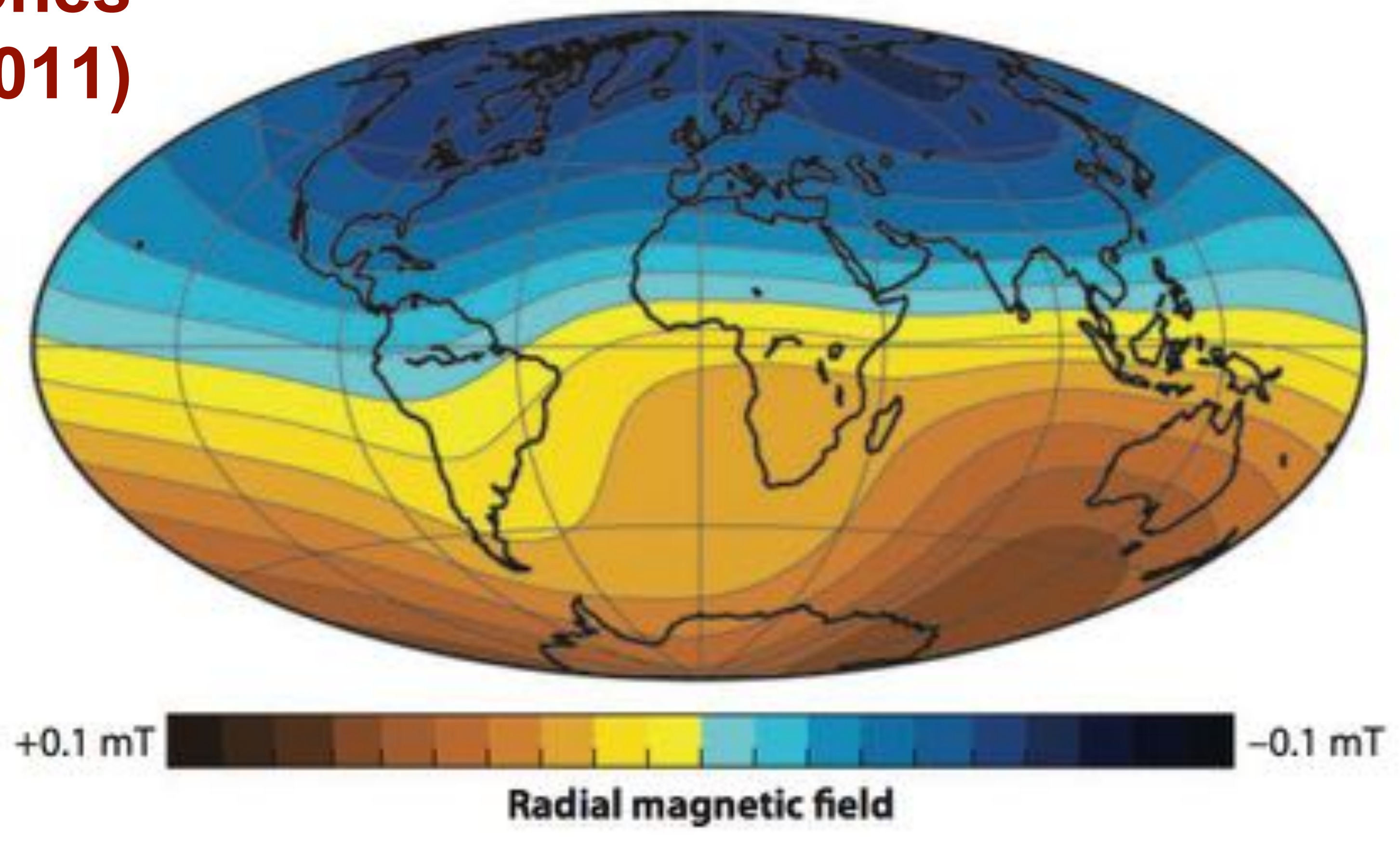
Earth

Magnetometer used by Alexander von Humboldt in his Latin America expedition of 1799-1804

Direct measurements of Earth's magnetic field date back to the early 1500's, with a boost in the early 1800's with the* Magnetic Crusade *led by Sabine in England and Gauss and Weber in Germany

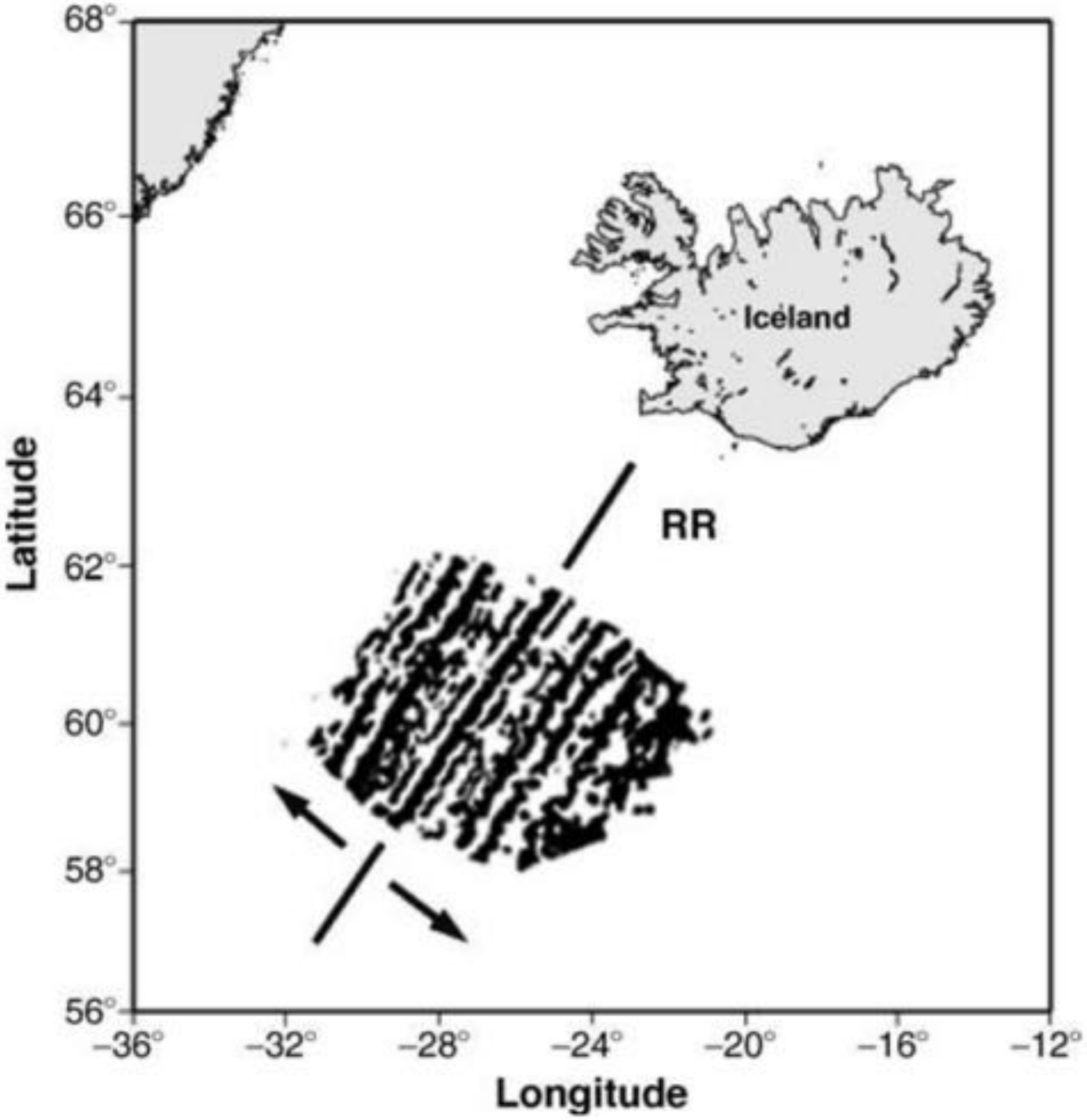
Today we also have satellite measurements

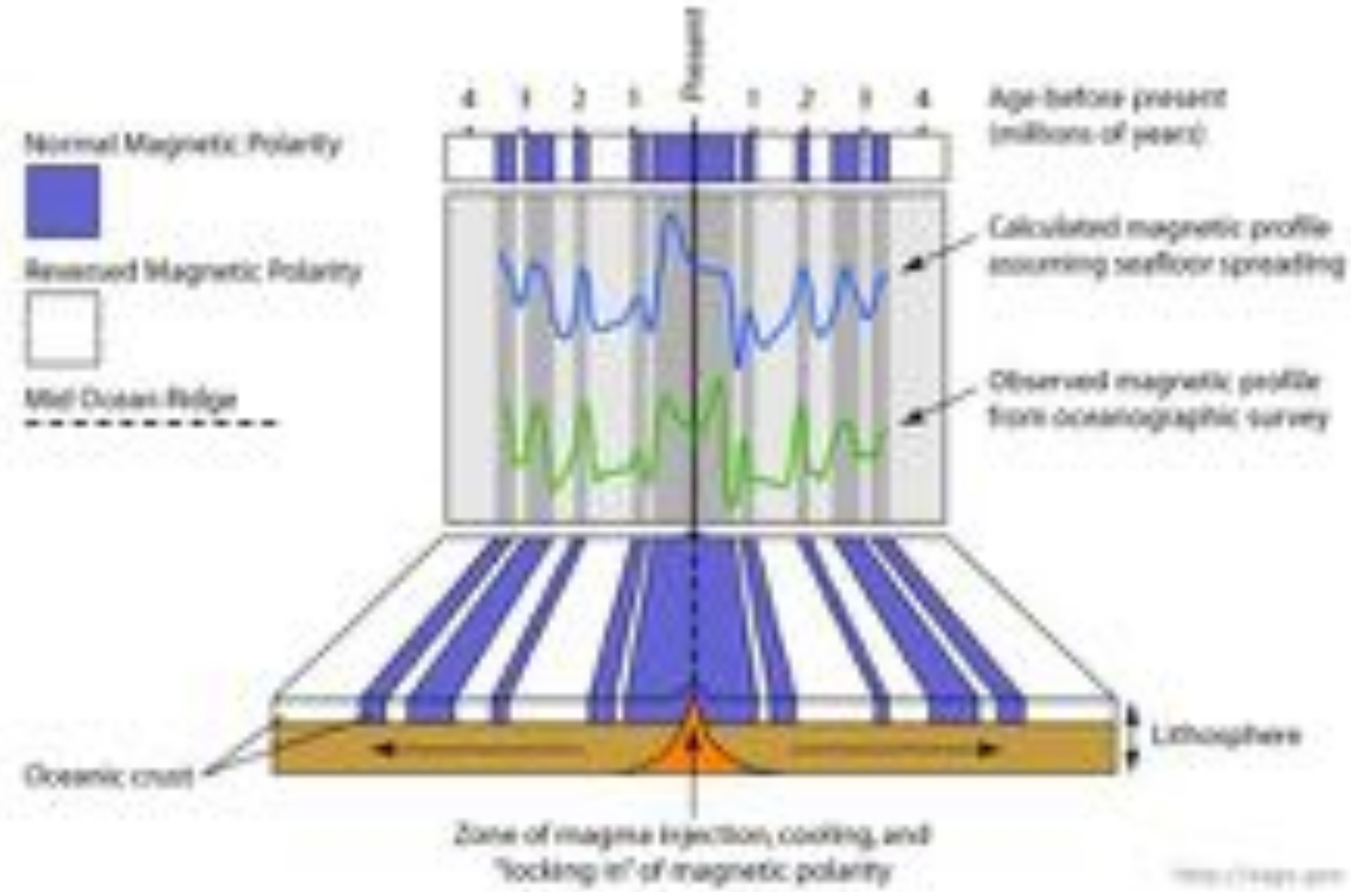
Jones (2011)



Longer time history can be inferred from measurements of magnetic signatures in crustal rocks

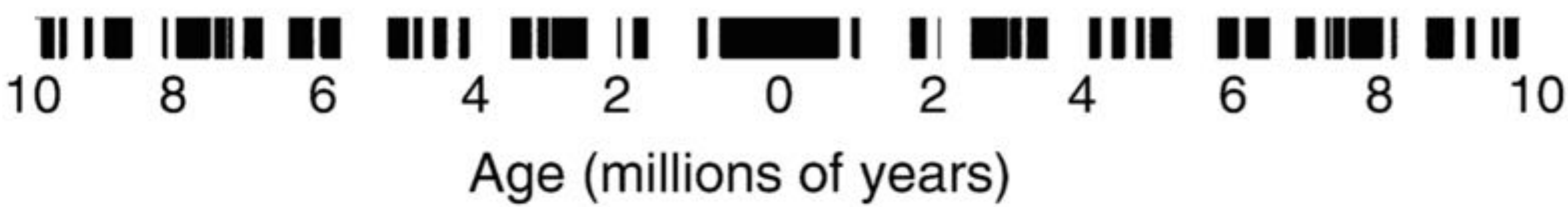
Heirtzler et al (1960's)



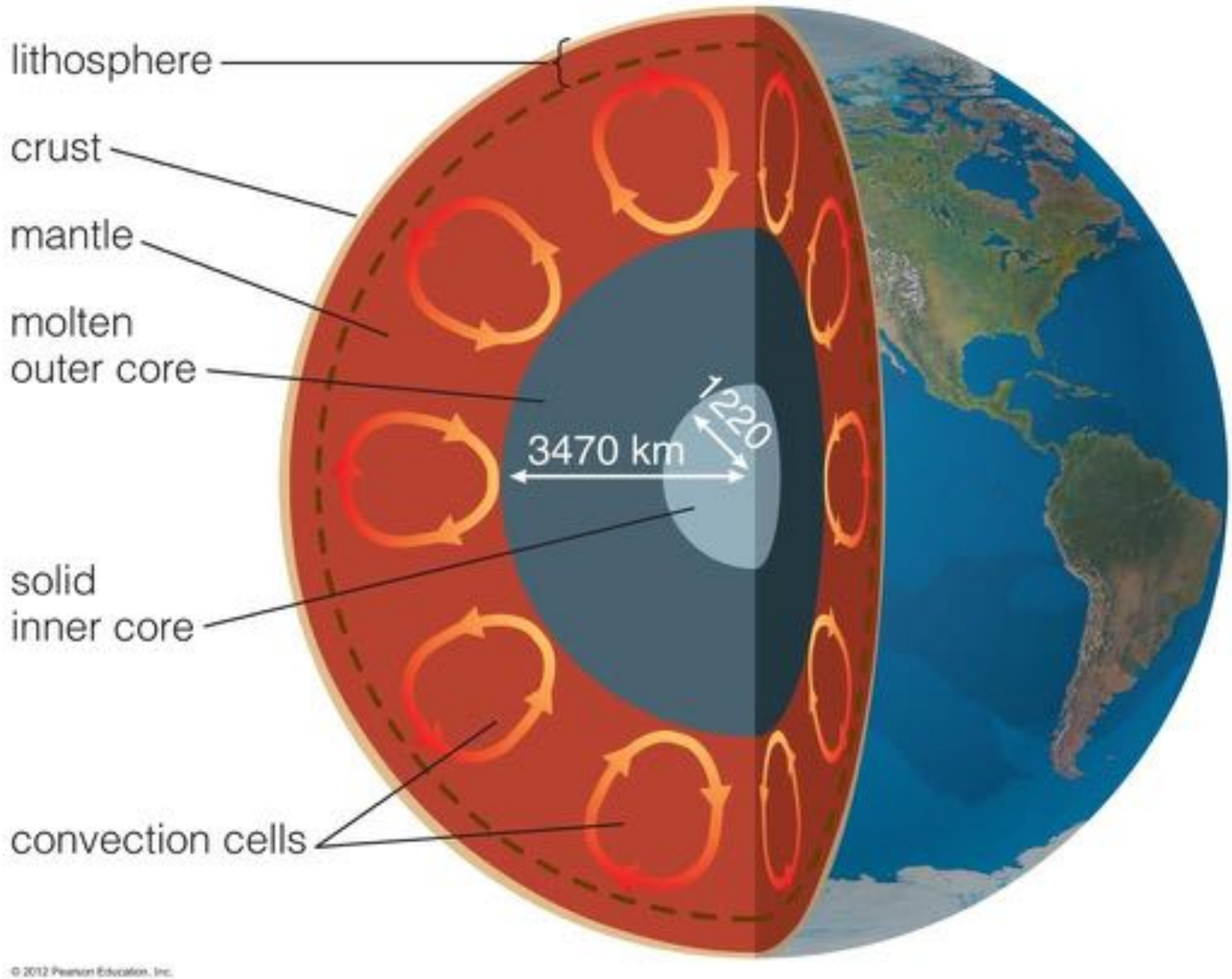


Magnetic poles flip every ~ 200,00 years on average, but randomly

Irregular reversals!



<u>Earth</u>



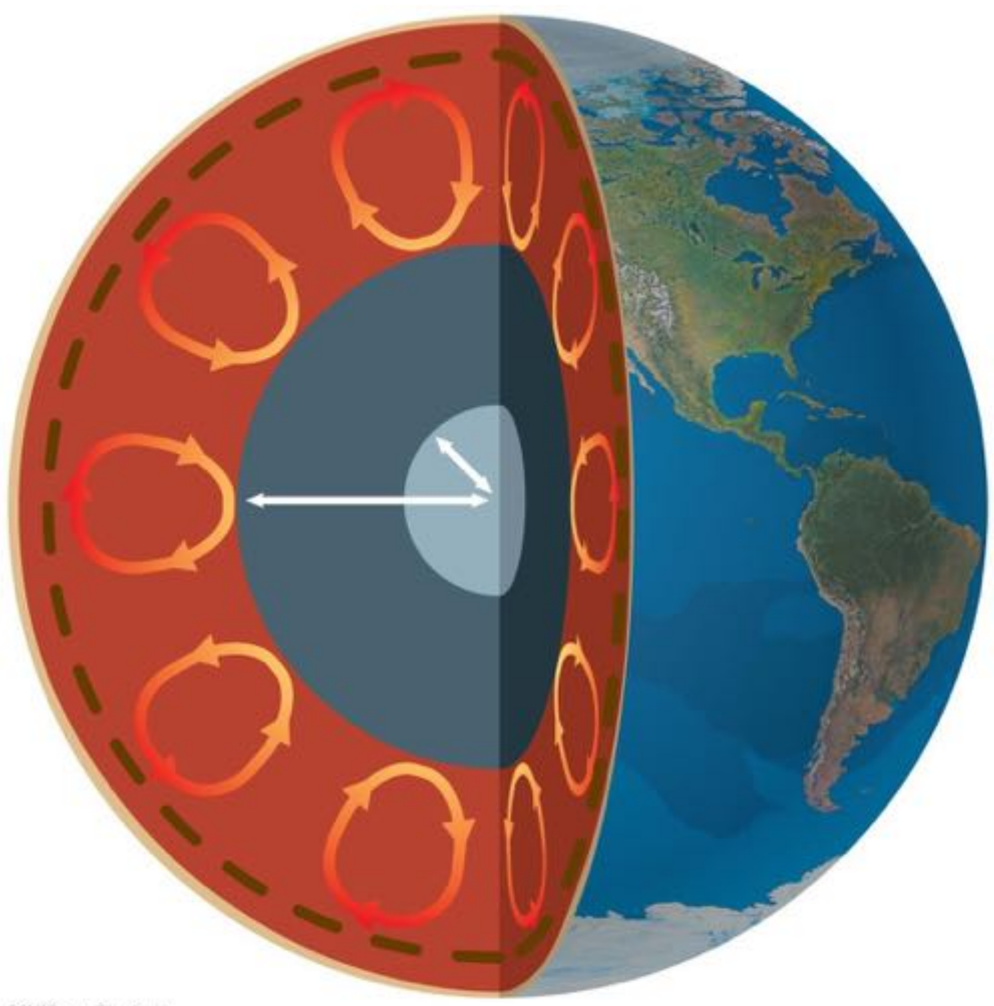
Mantle convection responsible for plate tectonics but not the geodynamo

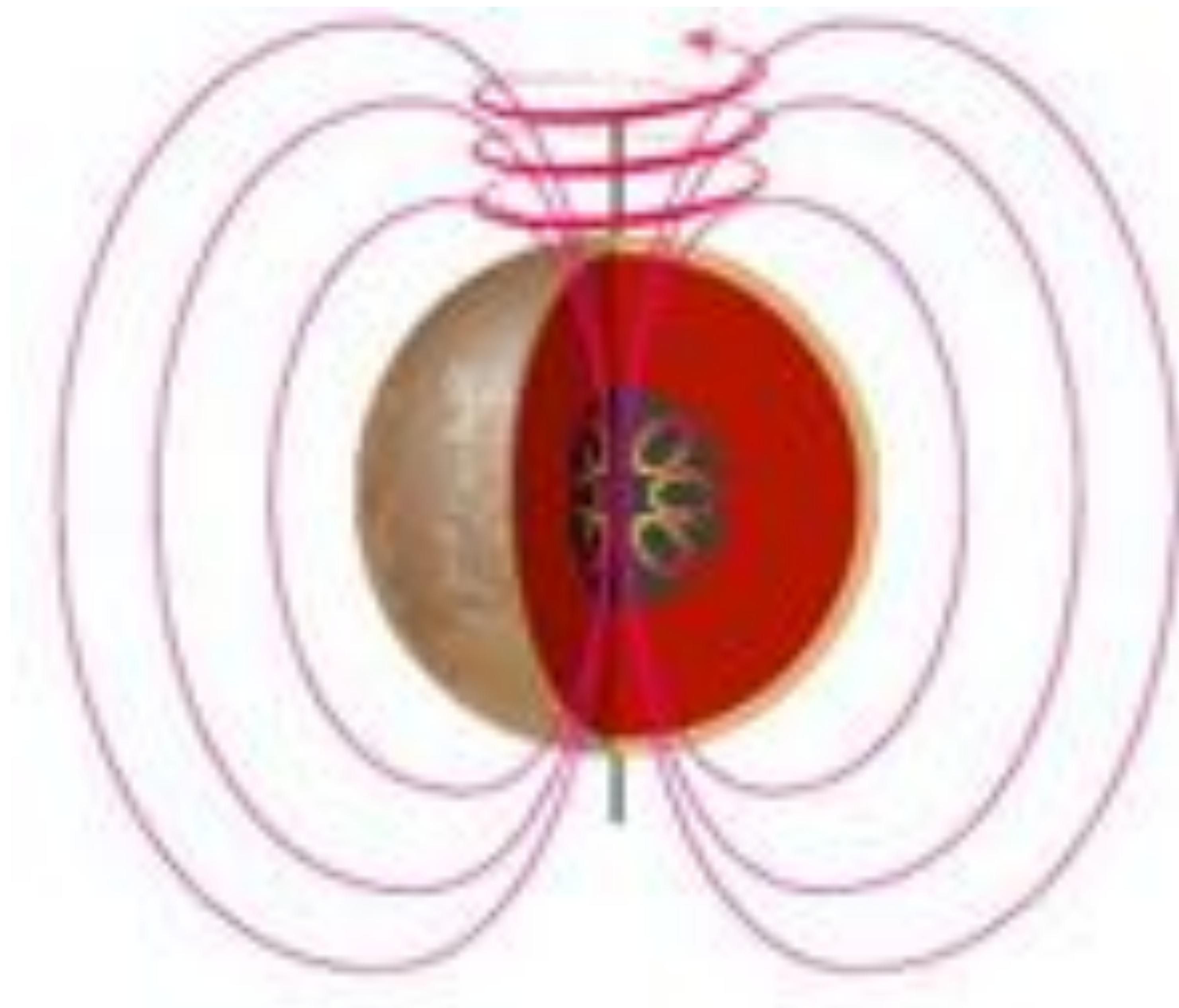
Why?

Earth

Mantle
non-conducting, slow

Overturning time
~100 million years





Outer Core
conducting, fast

Overturning time
~500 years

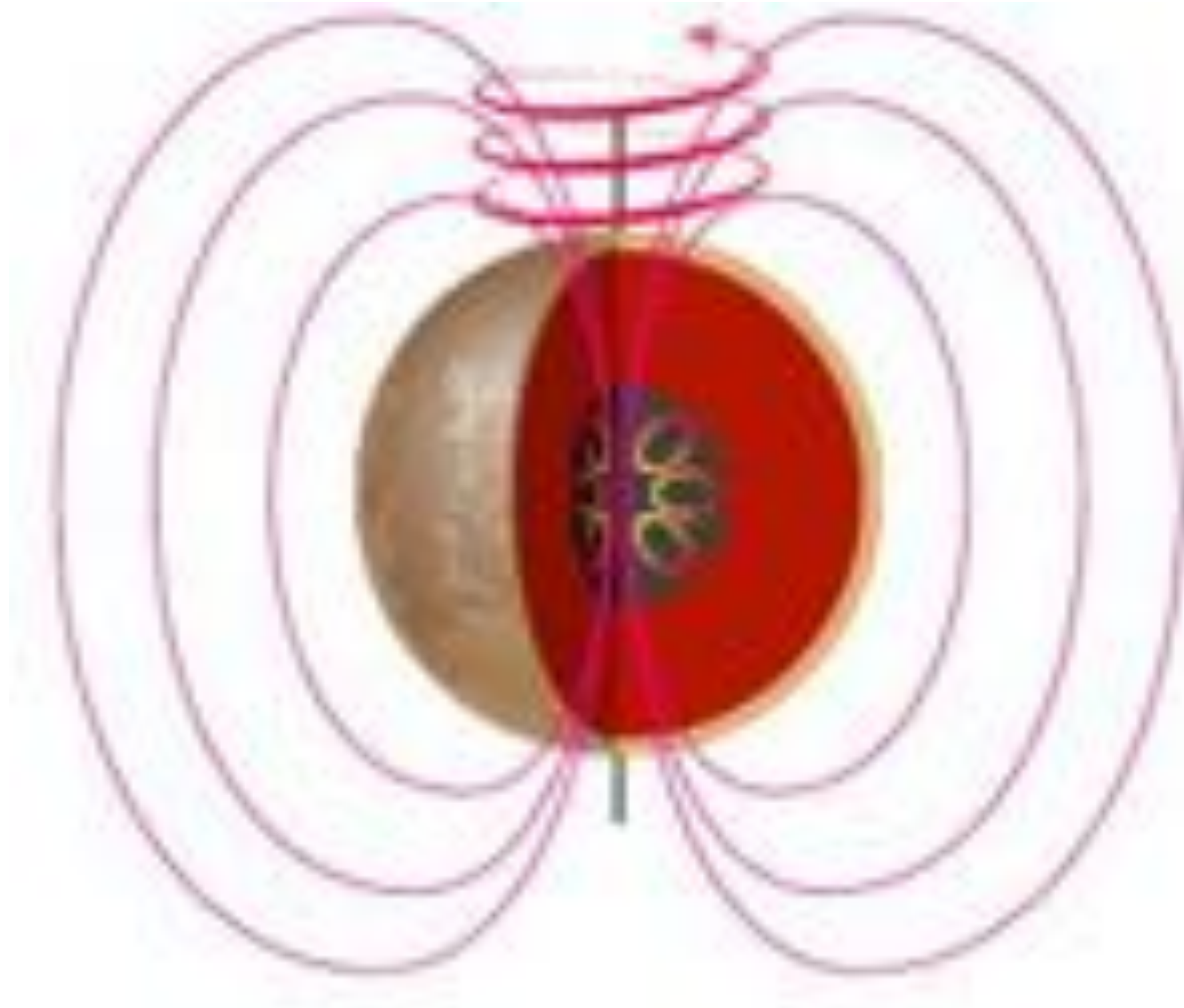
Earth

Rotational influence quantified by

Rossby number

$$\mathrm{Ro} = \frac{U}{2\Omega D} = \frac{1}{4\pi}\,\frac{P_{rot}}{\tau_c}$$

$$\mathrm{Ro} \sim 4 \times 10^{-7}$$



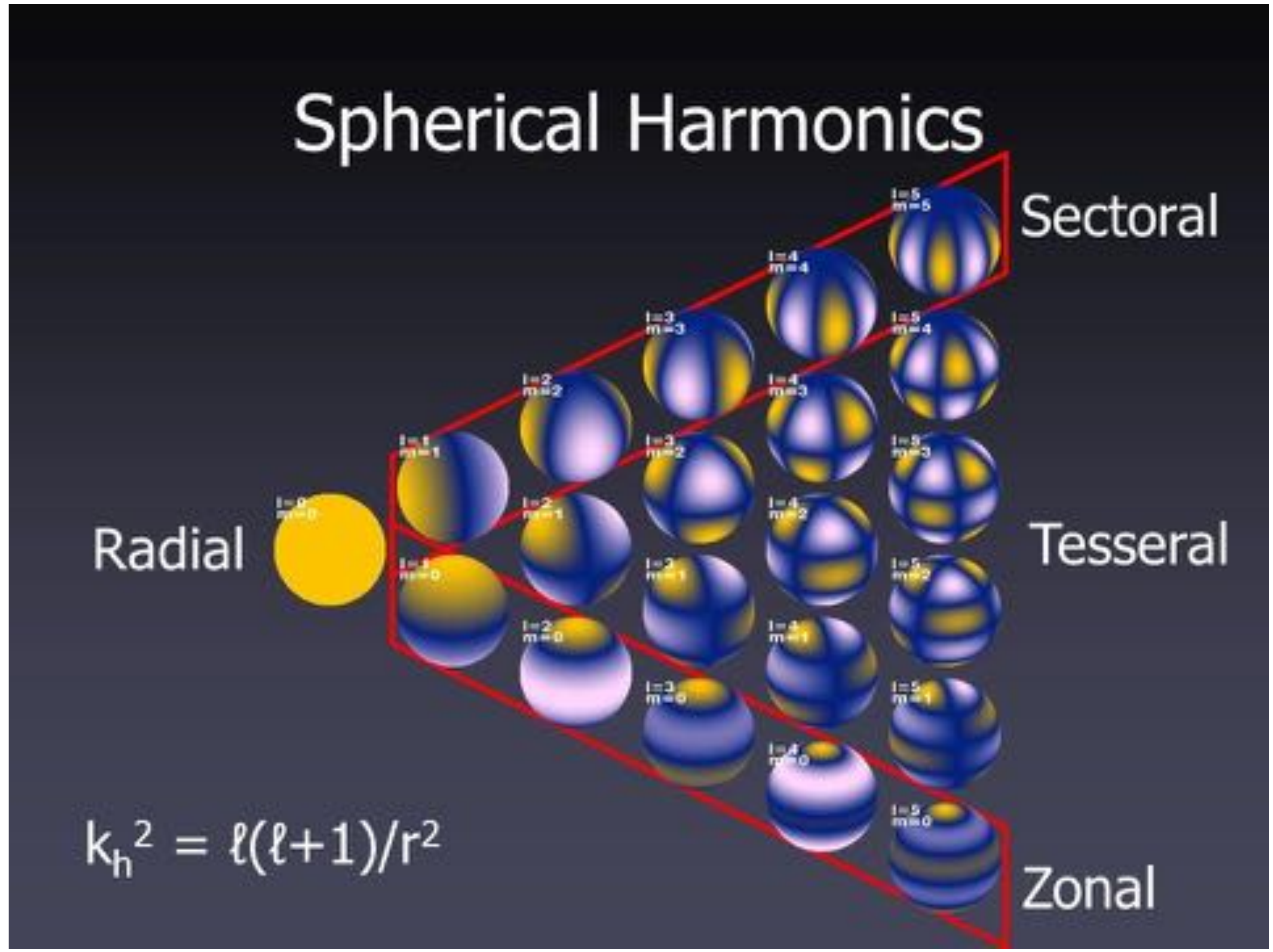
Outer Core
conducting, fast

Overturning time

~500 years

Earth

Spherical Harmonic expansion of the surface field allows for a backward extrapolation to the core-mantle boundary (CMB)



R. Townshend (Wisconsin)

Assuming no currents in the non-conducting mantle & crust

$$B_r \propto r^{-(\ell+2)}$$

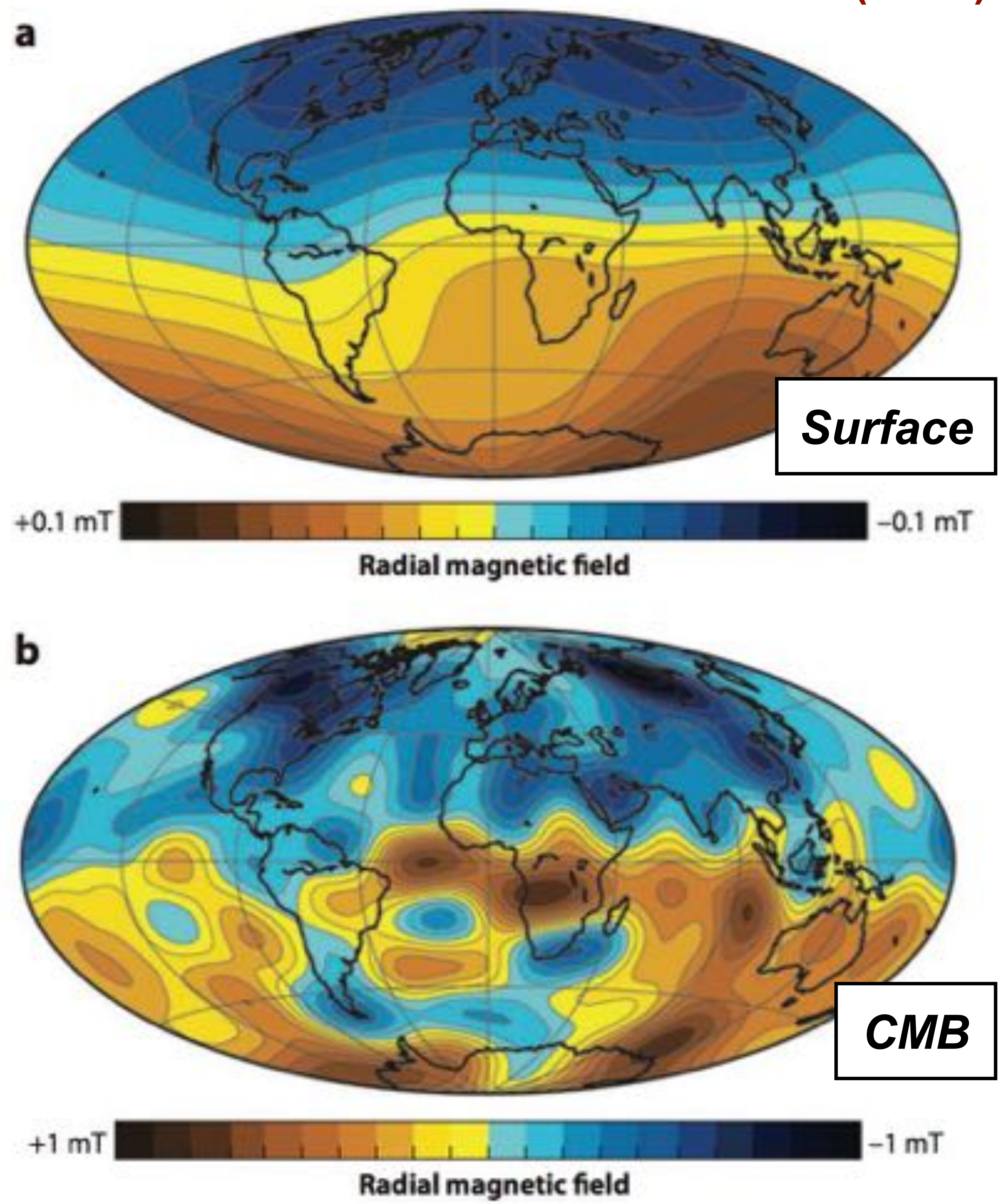
Jones (2011)

Earth

$$B_r \propto r^{-(\ell+2)}$$

Dipole dominates at large distances from the dynamo region ~ r^{-3}

Time evolution of surface field can be used to infer flows at the CMB



Earth

- **Energy sources for convective motions**
  - **Outward heat transport by conduction**
    - Cooling of the core over time
    - Proportional to the heat capacity
  - **Latent heat**
    - Associated with the freezing (phase change) of iron onto the solid core
  - **Gravitational Differentiation**
    - Redistribution of light and heavy elements, releasing gravitational potential energy
  - **Radioactive Heating**
    - Energy released by the decay of heavy elements

Venus

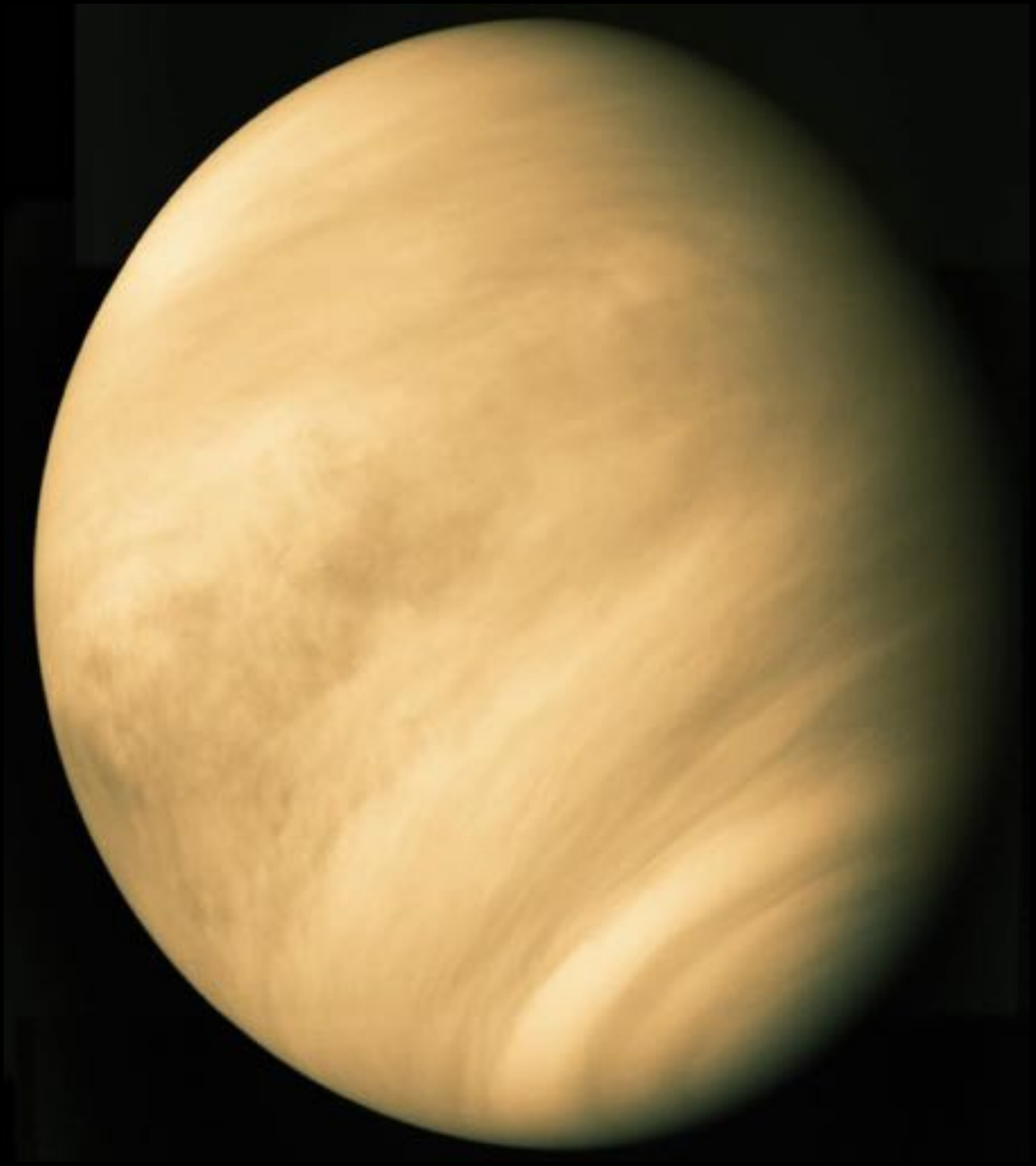
No Dynamo

No field detected

Why?

Core may be liquid and conducting, but it may not be convecting (rigid top may inhibit cooling)

Also - slow rotation



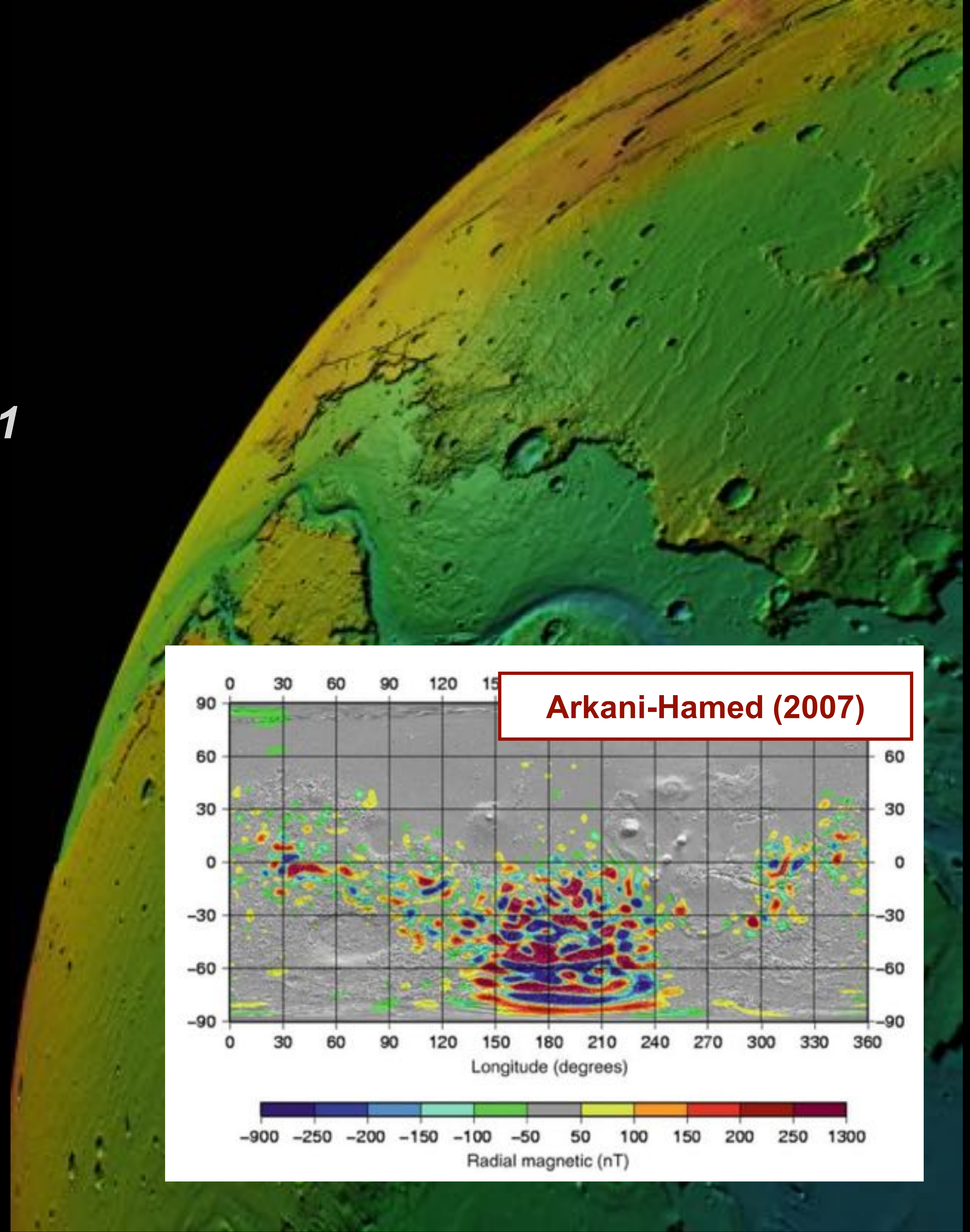
Mars

No Dynamo

Fields patchy, reaching ~ 0.01 G in spots but no dipole

Why?

It had a dynamo in the past (remnant crustal magnetism) but it cooled off fast, freezing out its molten core



Mercury

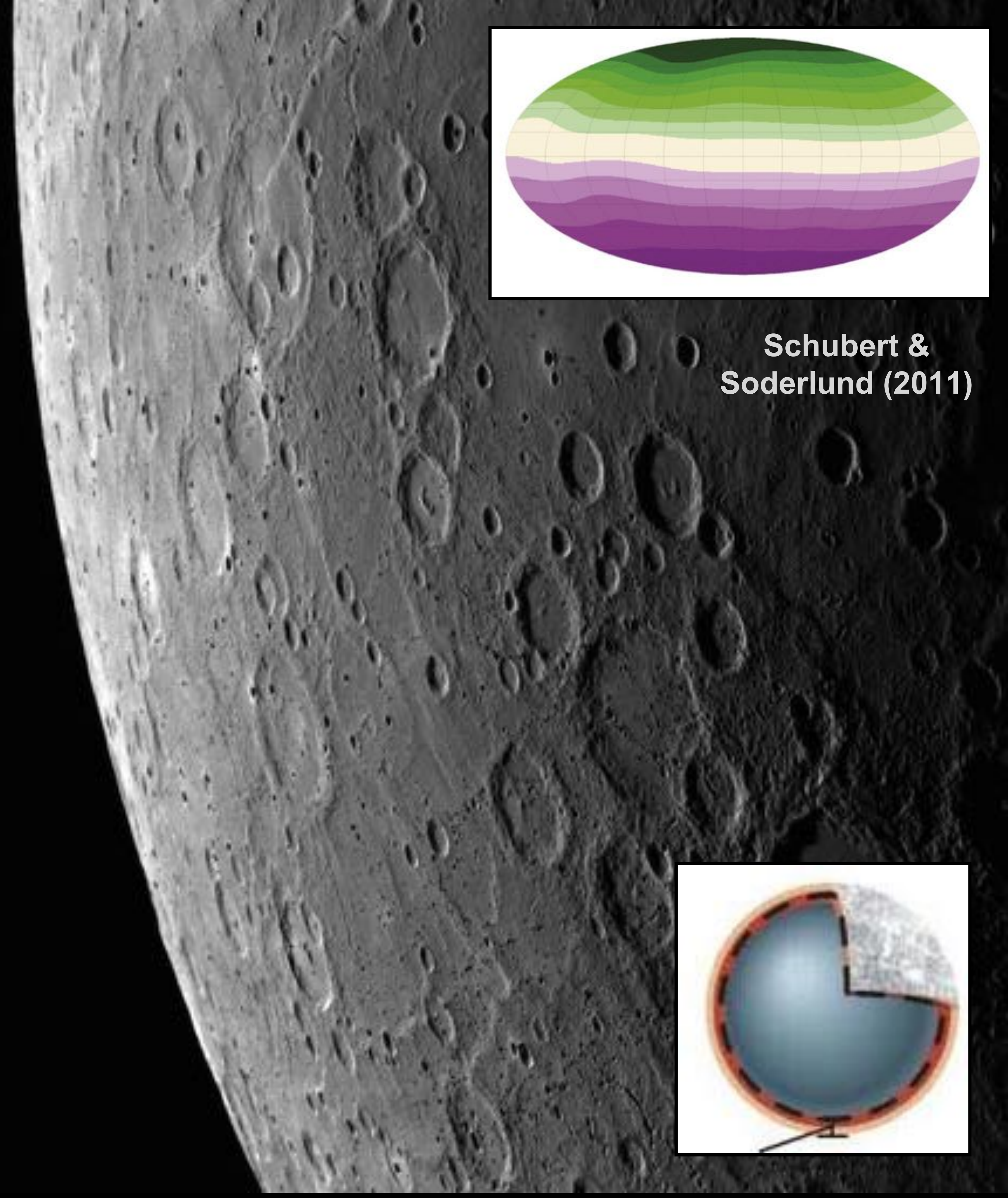
Dynamo!

Field strength
~ 0.003 G

Dipolarity
~ 0.71 G

Tilt ~ 3°

Huge iron core relative to size of planet that is still partially molten



Schubert & Soderlund (2011)

Ganymede!

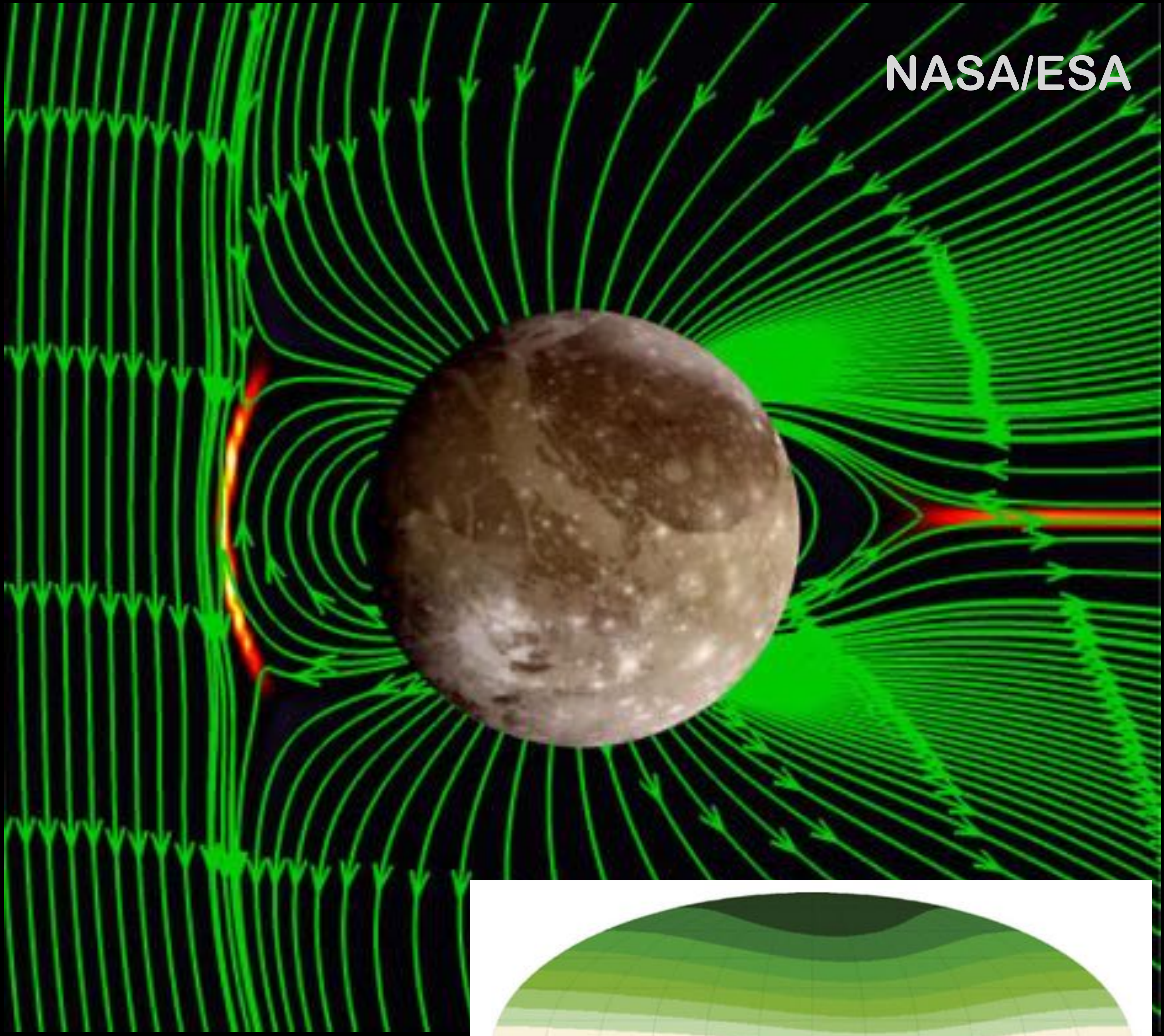
Dynamo!

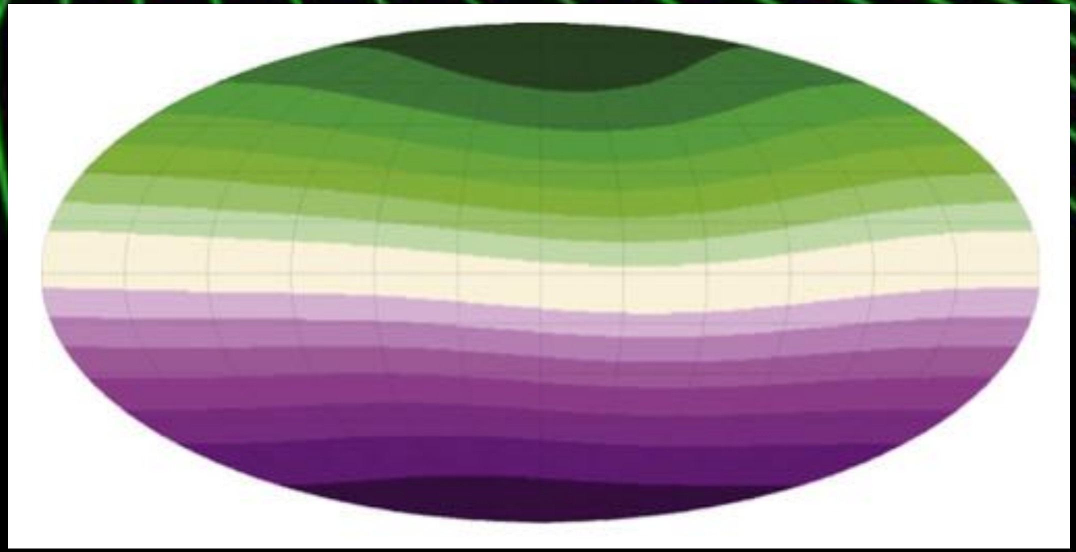
Field strength
~ 0.01 G

Dipolarity
~ 0.95 G

Tilt
~ 4°

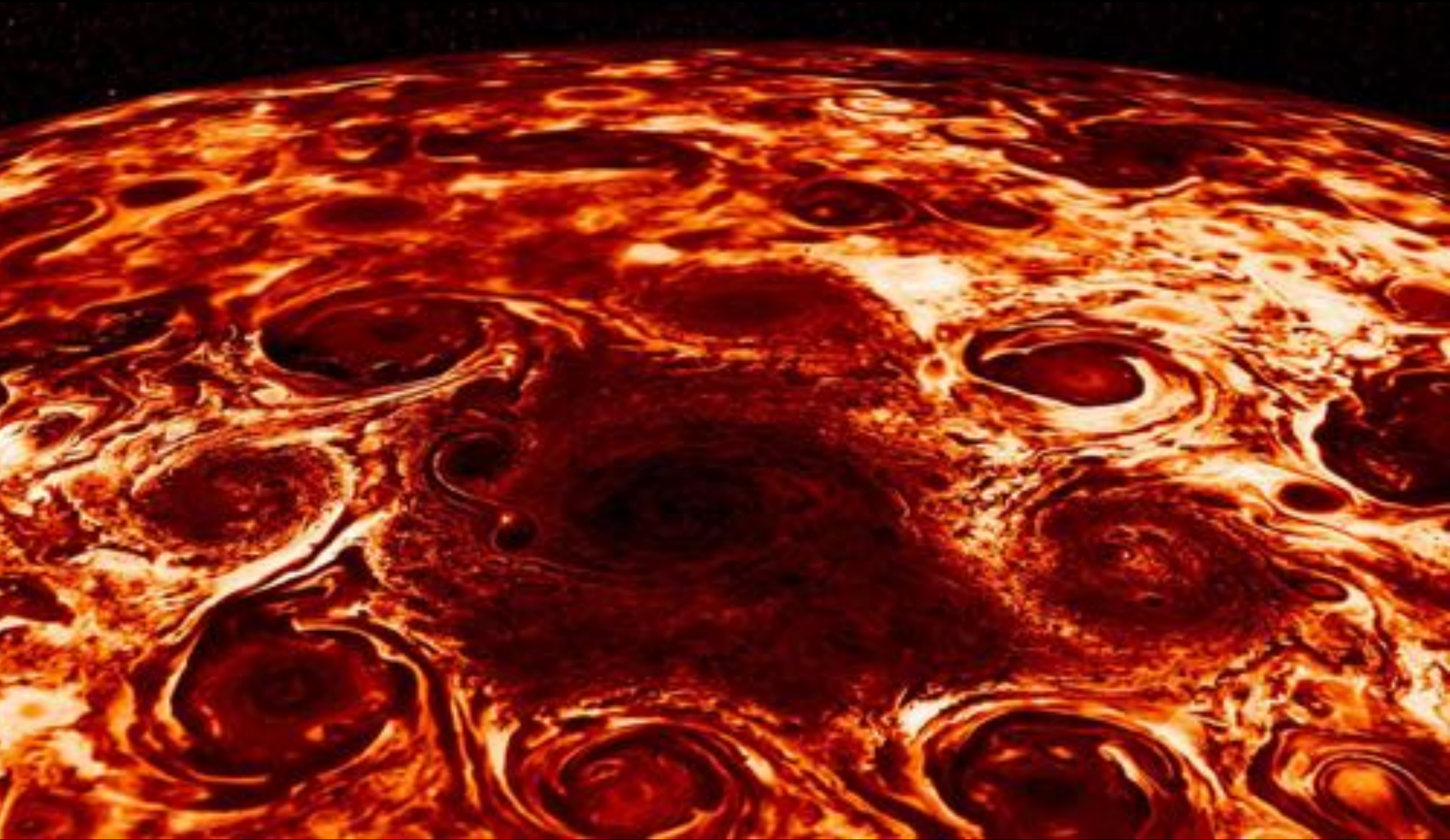
Other icy satellites have induced magnetic fields from passing through the magnetospheres of their planets





Schubert & Soderlund (2011)

Juno!

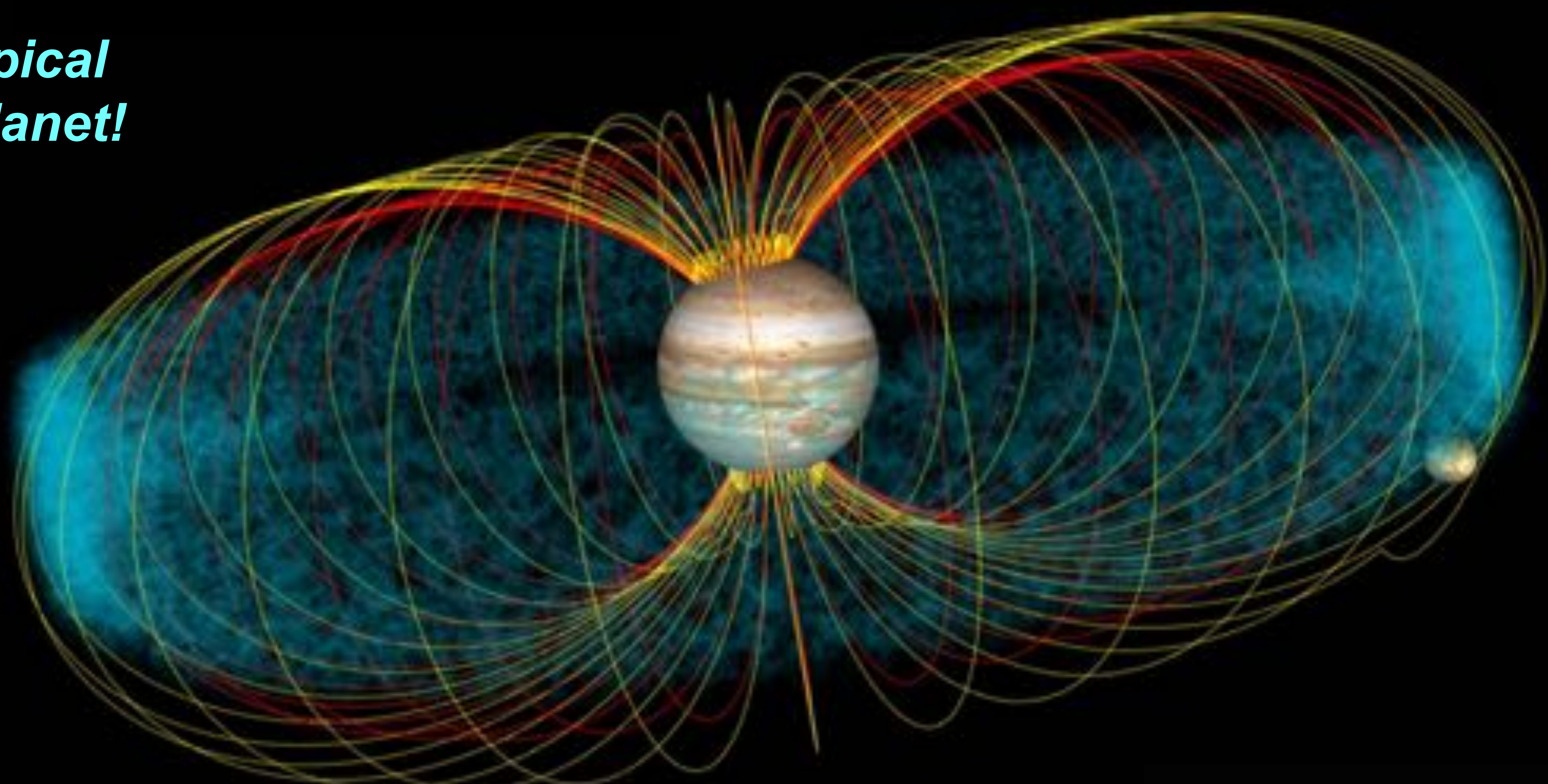


Jupiter

Big Whopping Dynamo!

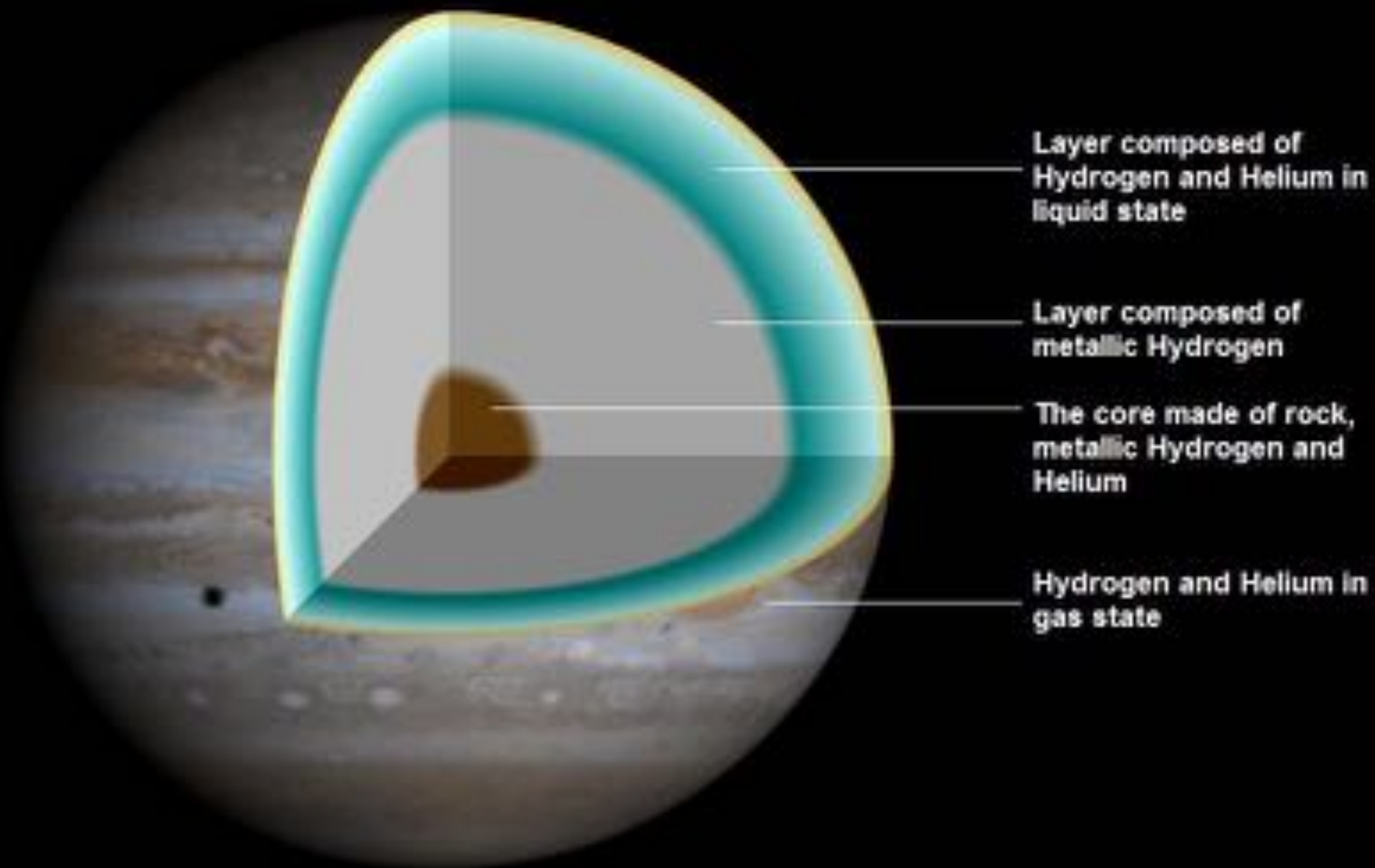
Field strength ~ 7 G *Dipolarity ~ 0.61* *Tilt ~ 10°*

Archetypical Jovian planet!



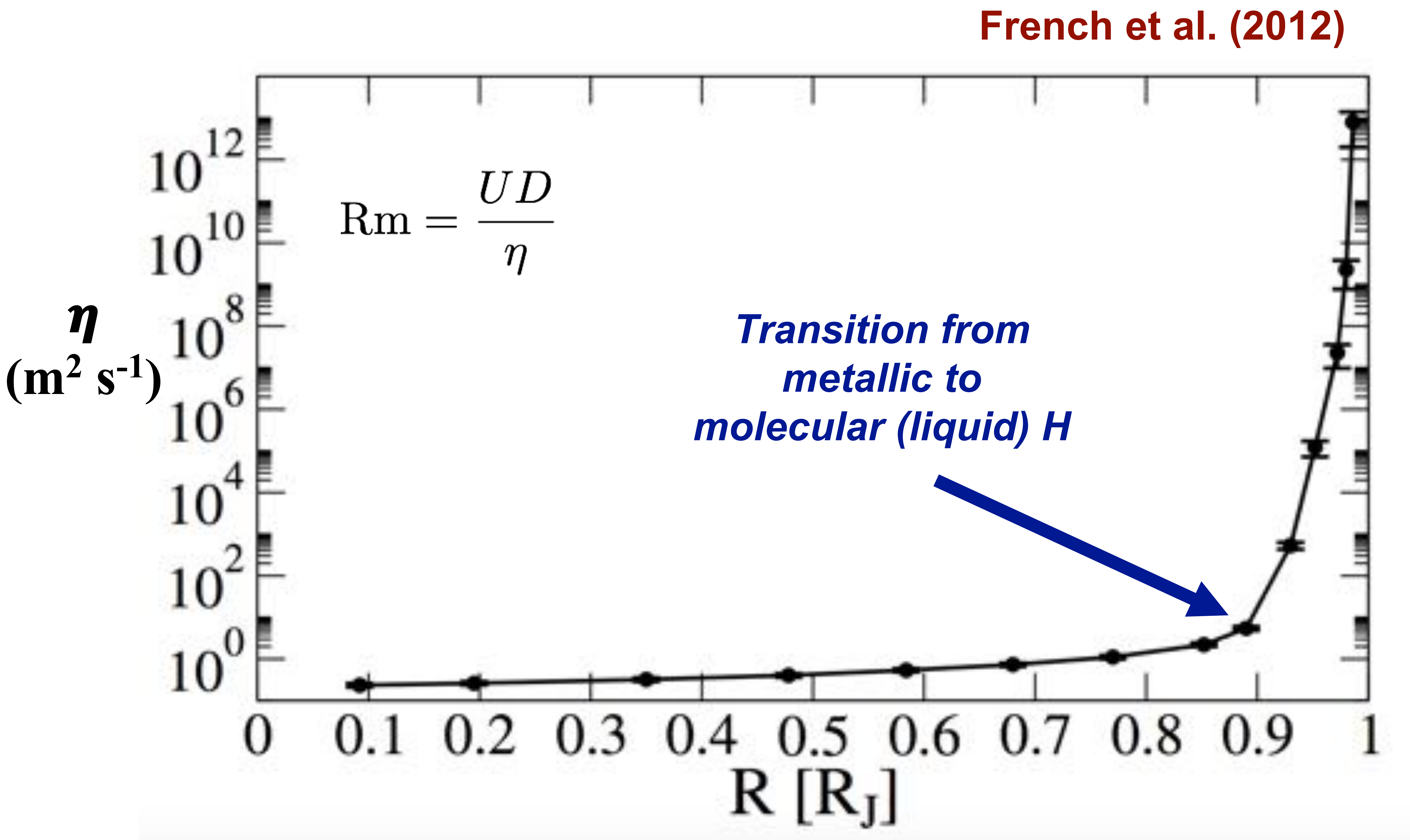
Goddard Space Flight Center

Jupiter

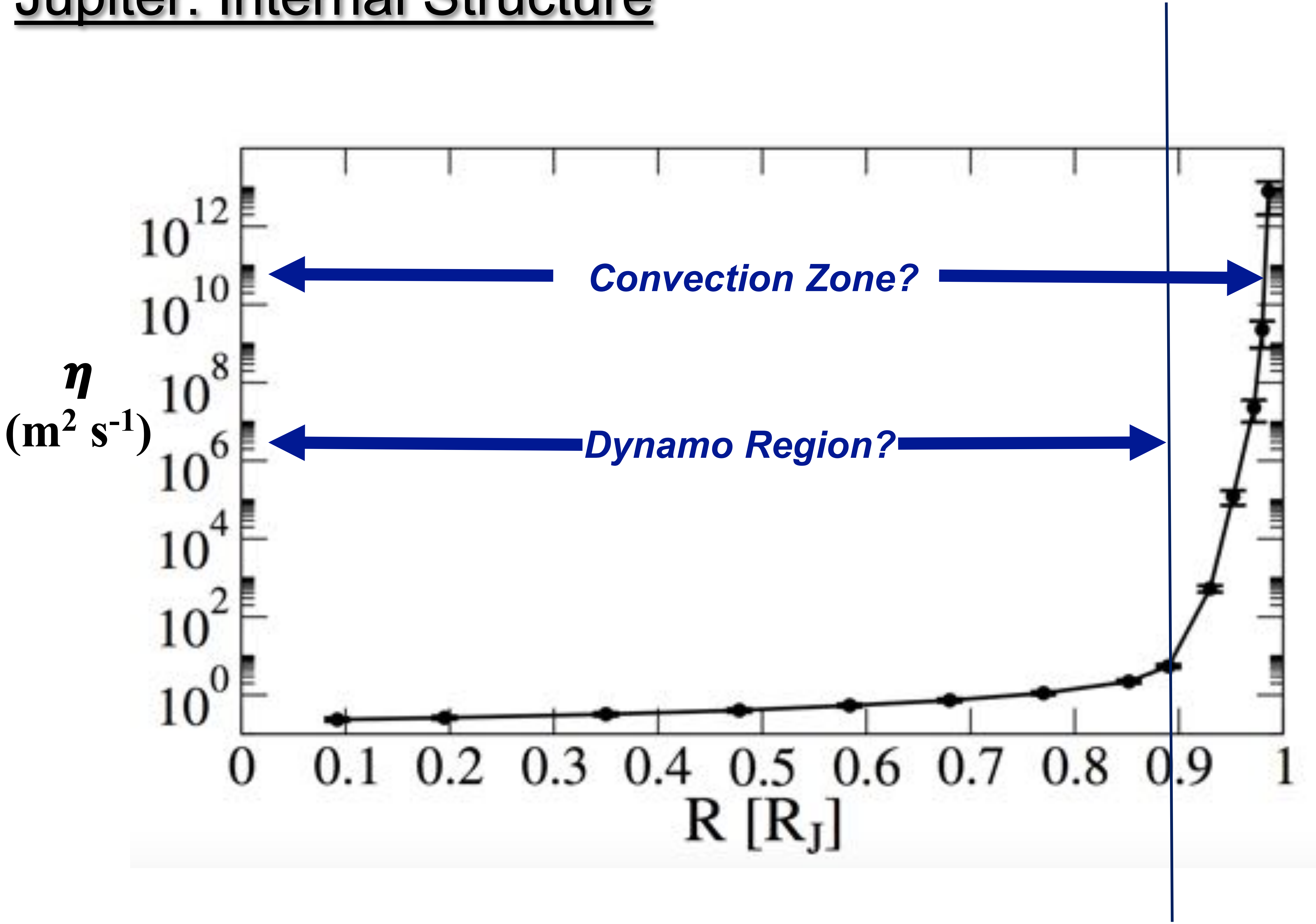


Layer composed of Hydrogen and Helium in liquid state
Layer composed of metallic Hydrogen
The core made of rock, metallic Hydrogen and Helium
Hydrogen and Helium in gas state

Jupiter: Internal Structure

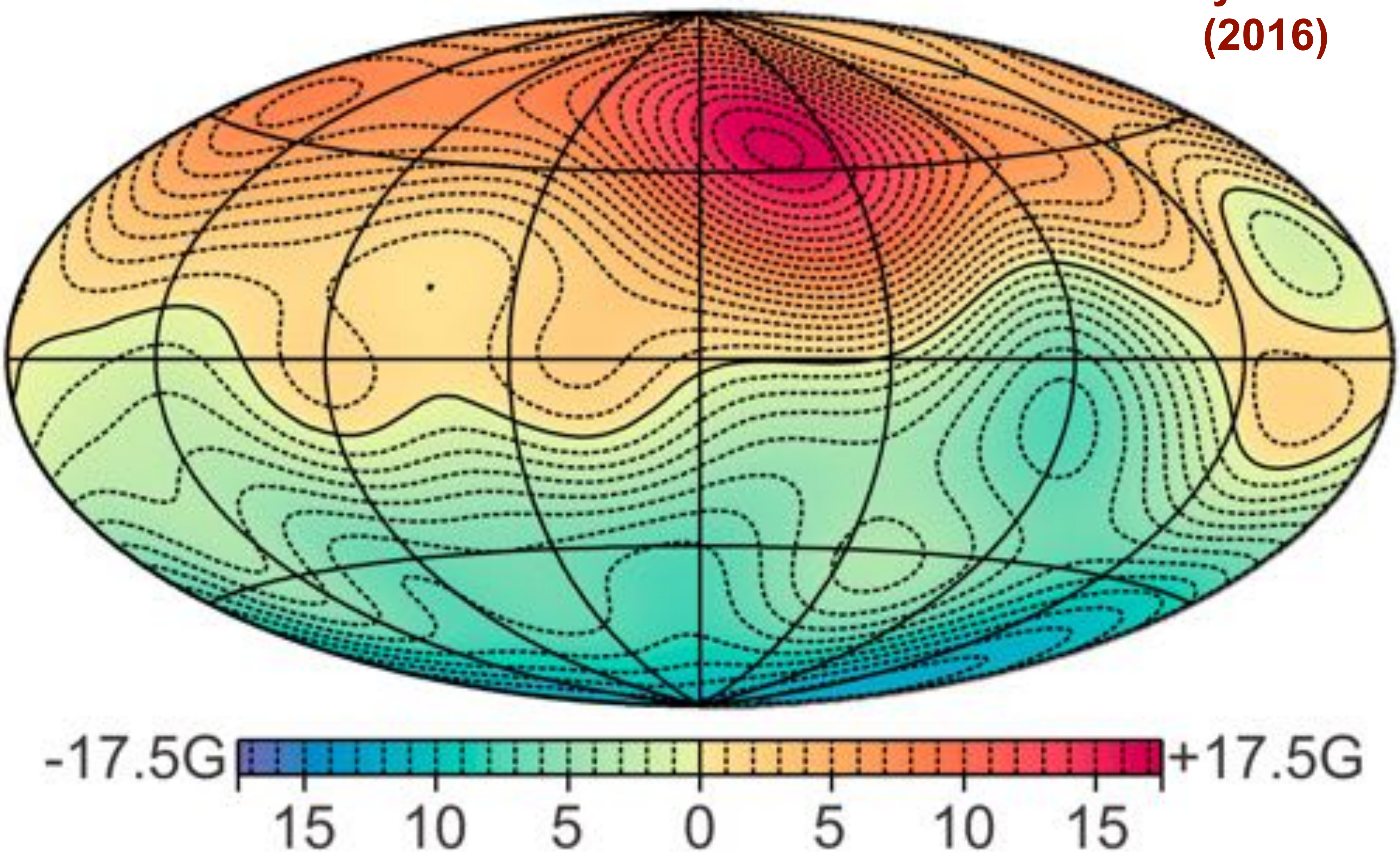


Jupiter: Internal Structure



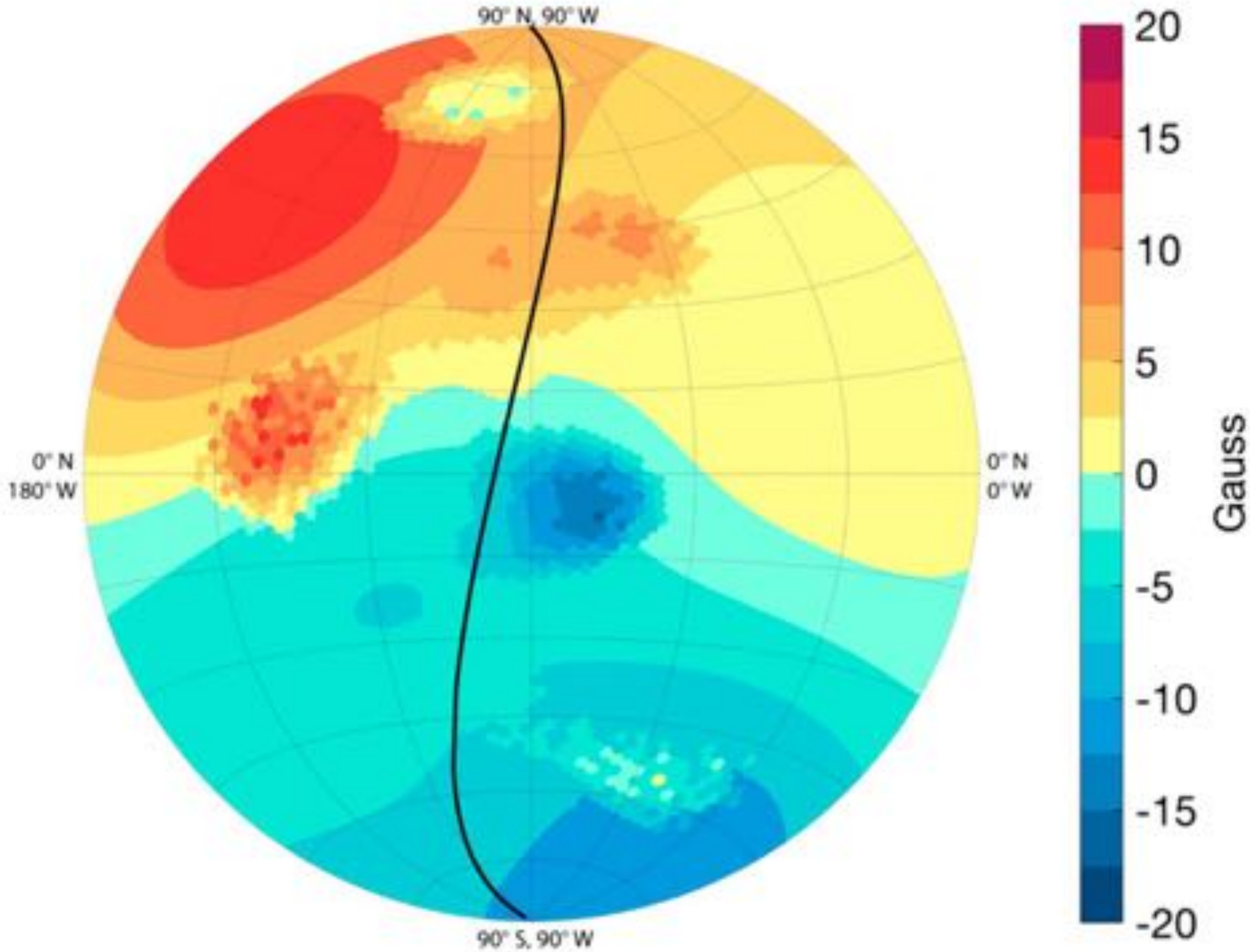
Jupiter: Magnetic Field (Pre-Juno)

Ridley & Holme (2016)



Initial results from Juno

Stronger and more patchy than expected (higher-order multipoles)



$$B_r \propto r^{-(\ell+2)}$$

Moore et al (2017)

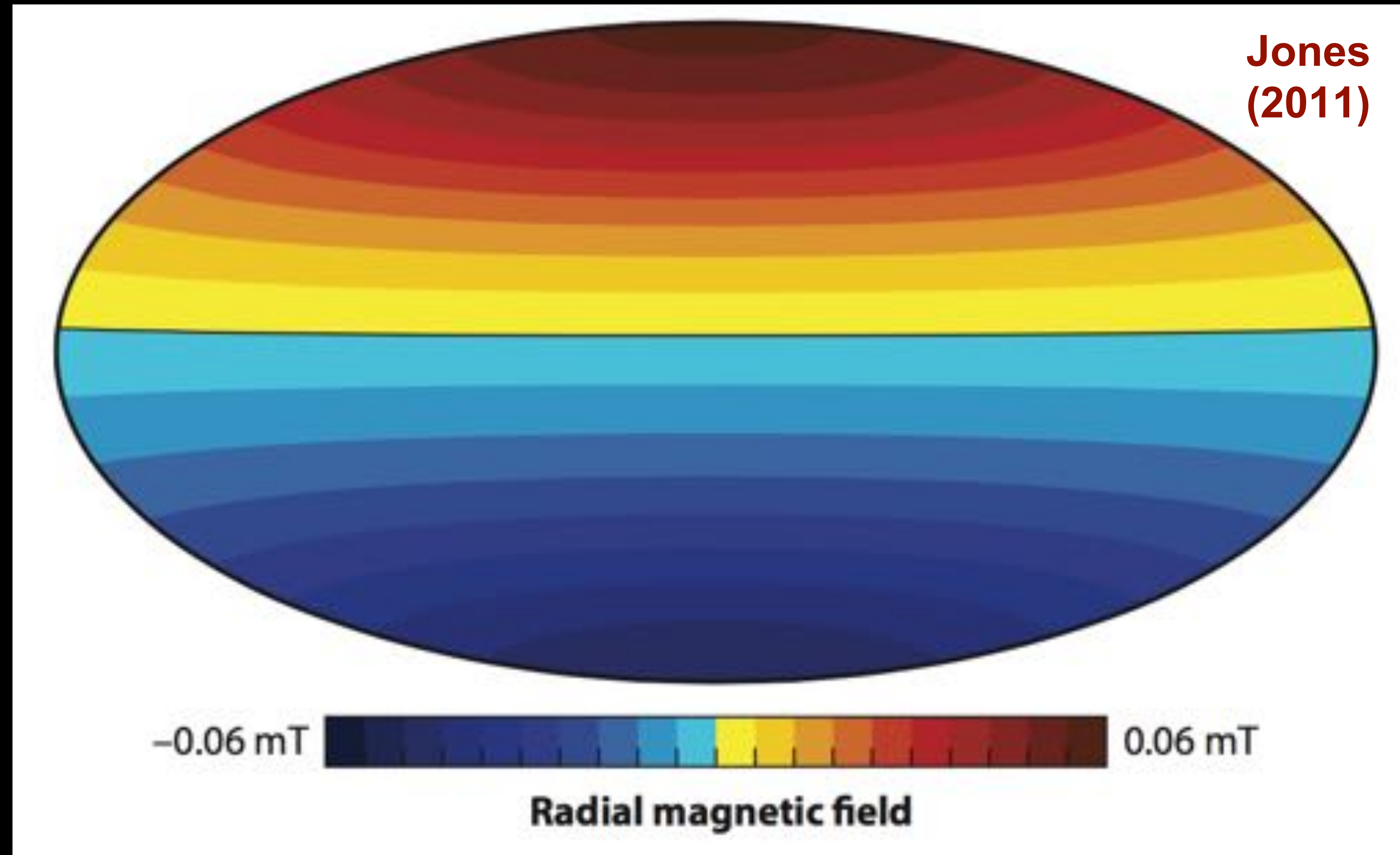
Saturn

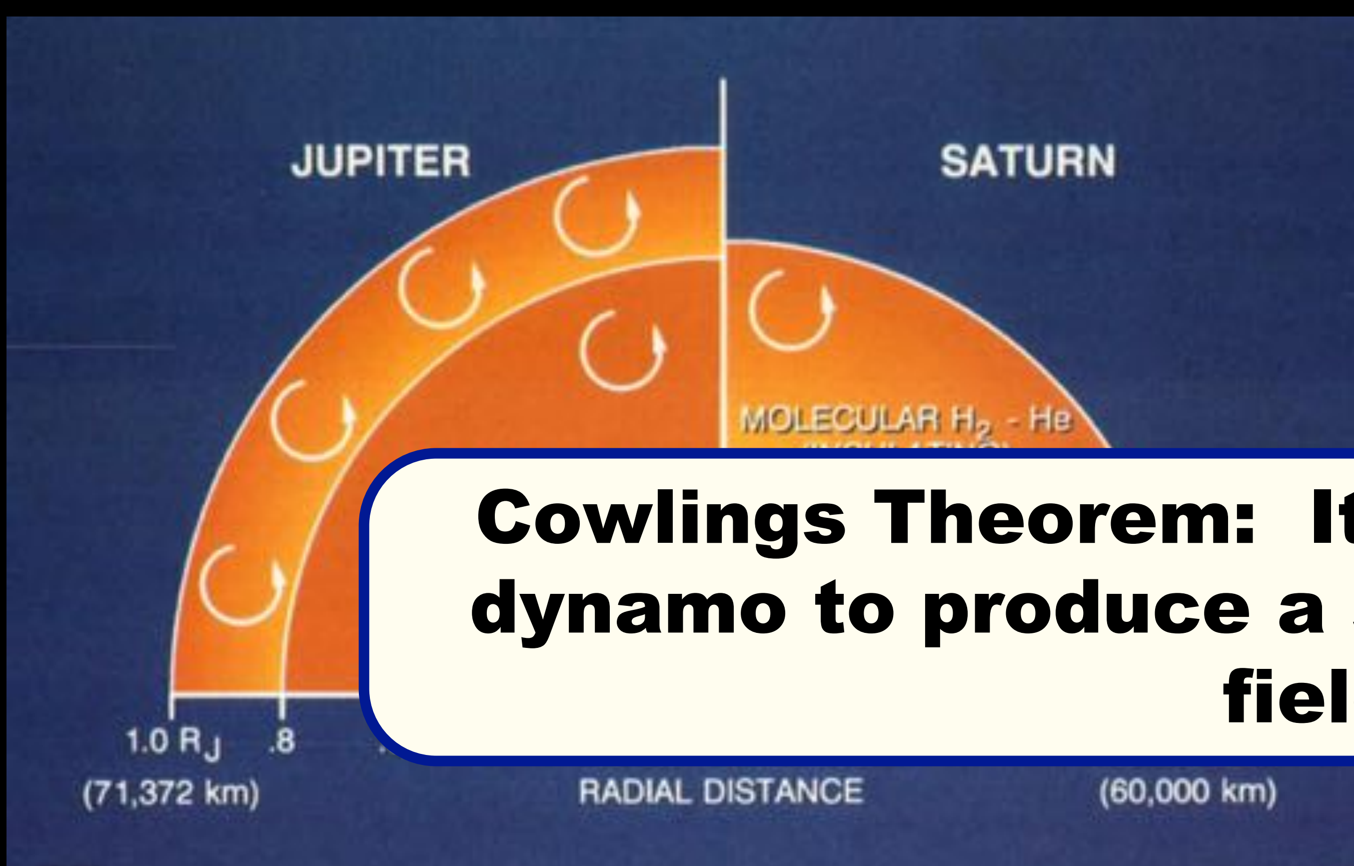
Dynamo!

Field strength
~ 0.6 G

Dipolarity
~ 0.85 G

Tilt
< 0.5°





Remarkably axisymmetric!

A surprise!

Cowlings Theorem: It is not possible for a dynamo to produce a steady axisymmetric field!!

Connerney(1993)

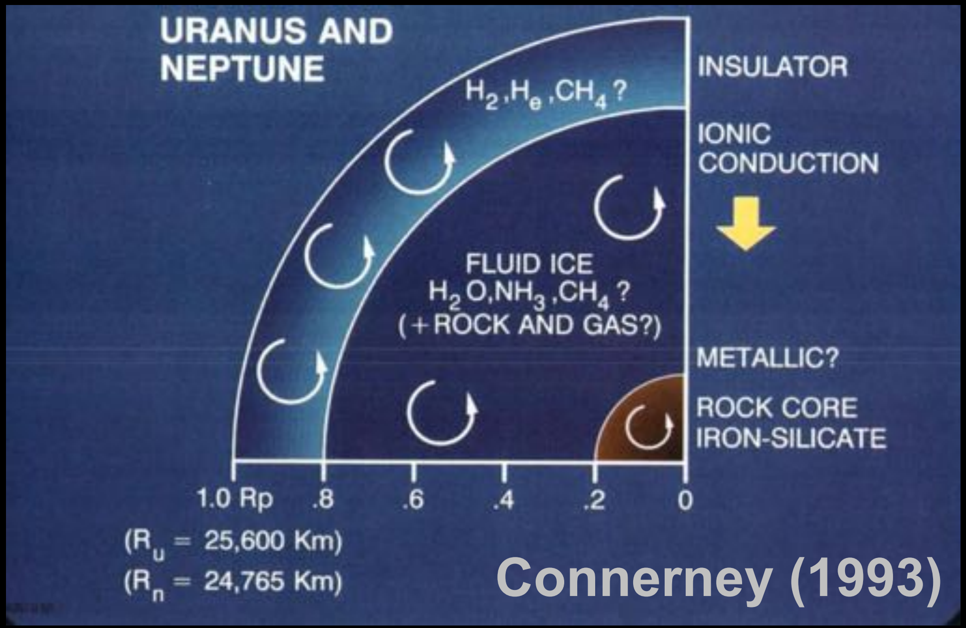
Uranus & Neptune

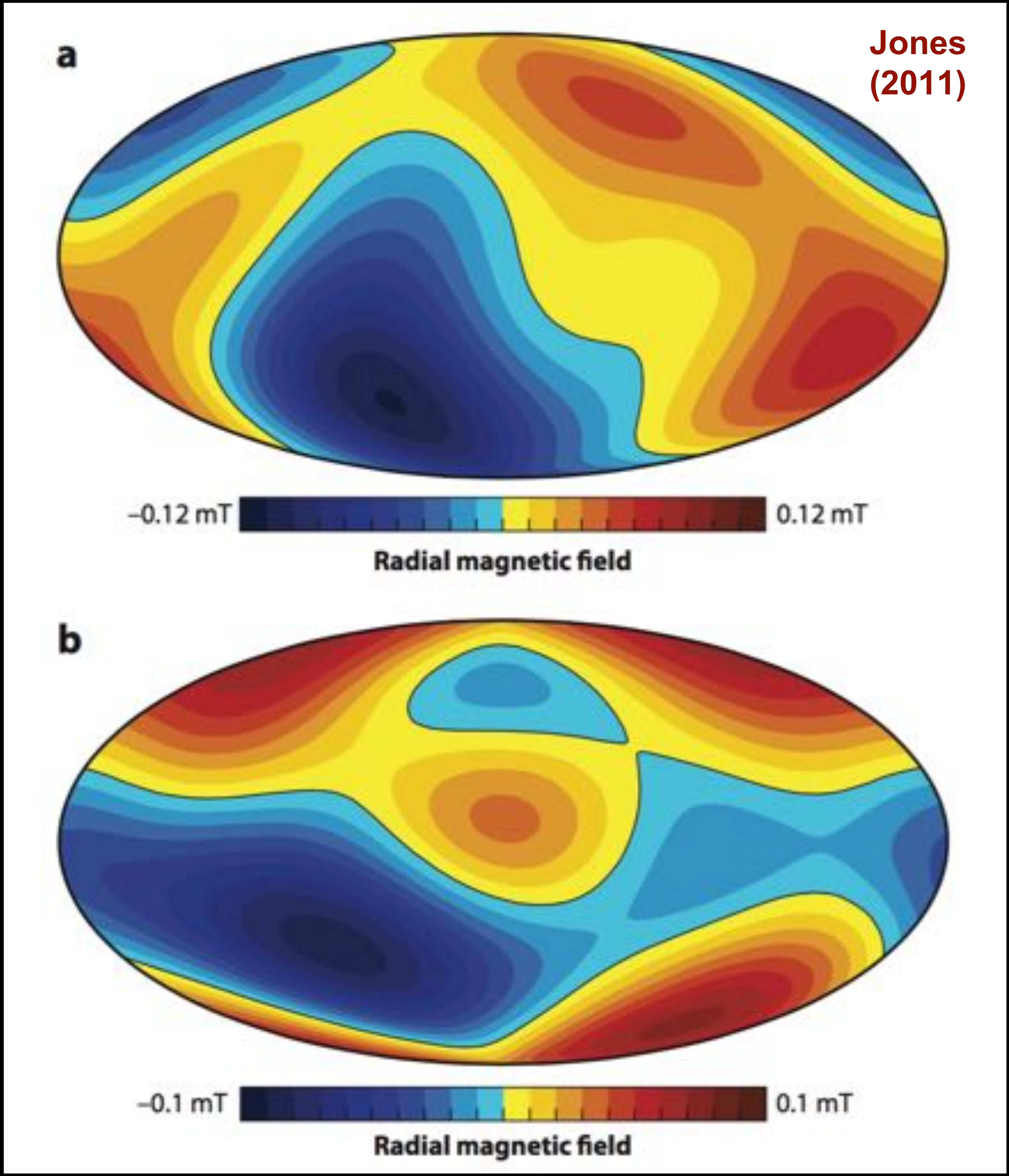
Dynamos!

Field strength ~ 0.3 G

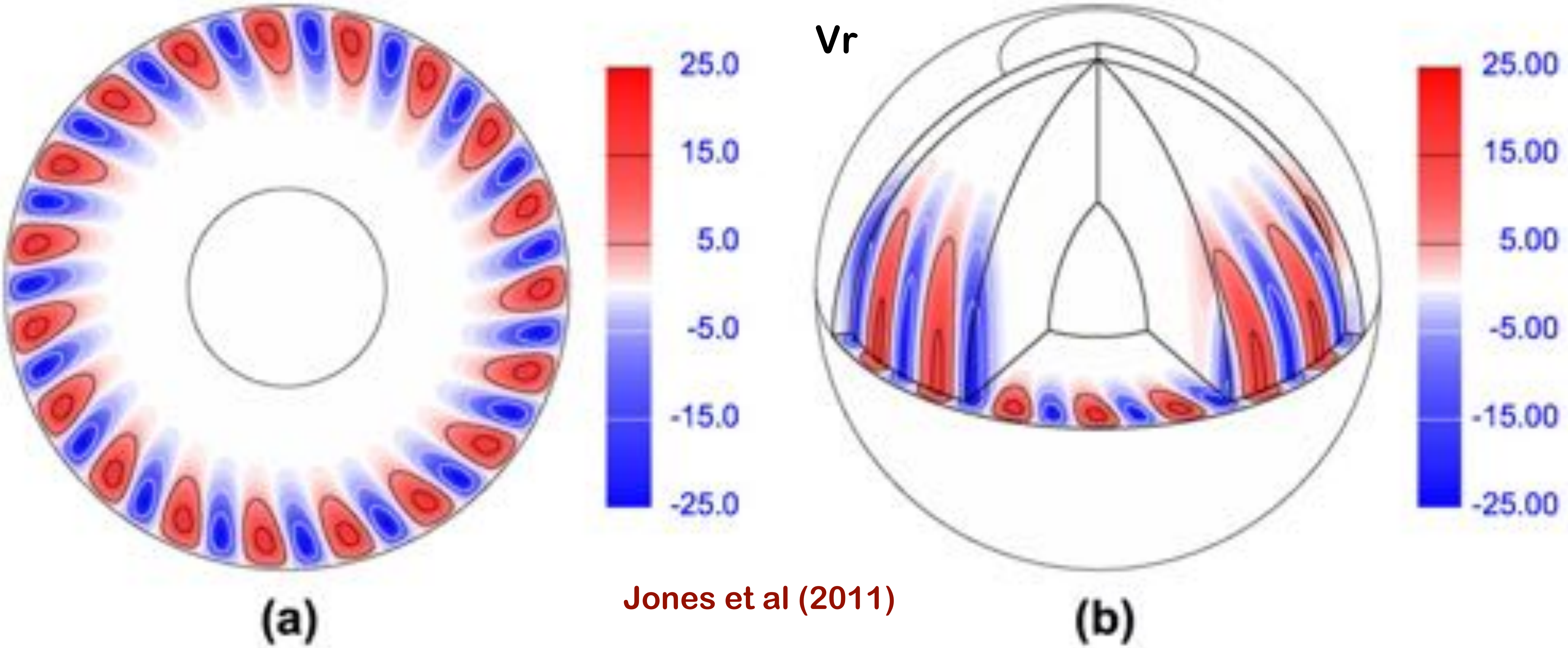
Dipolarity ~ 0.42, 0.31

Tilt ~ 59°, 45°





Nonlinear Regimes require Numerical Models



Jones et al (2011)

Solve the MHD equations in a rotating spherical shell
Anelastic or Boussinesq approximation
ρ, T, P, S are linear perturbations about a
hydrostatic, adiabatic background state

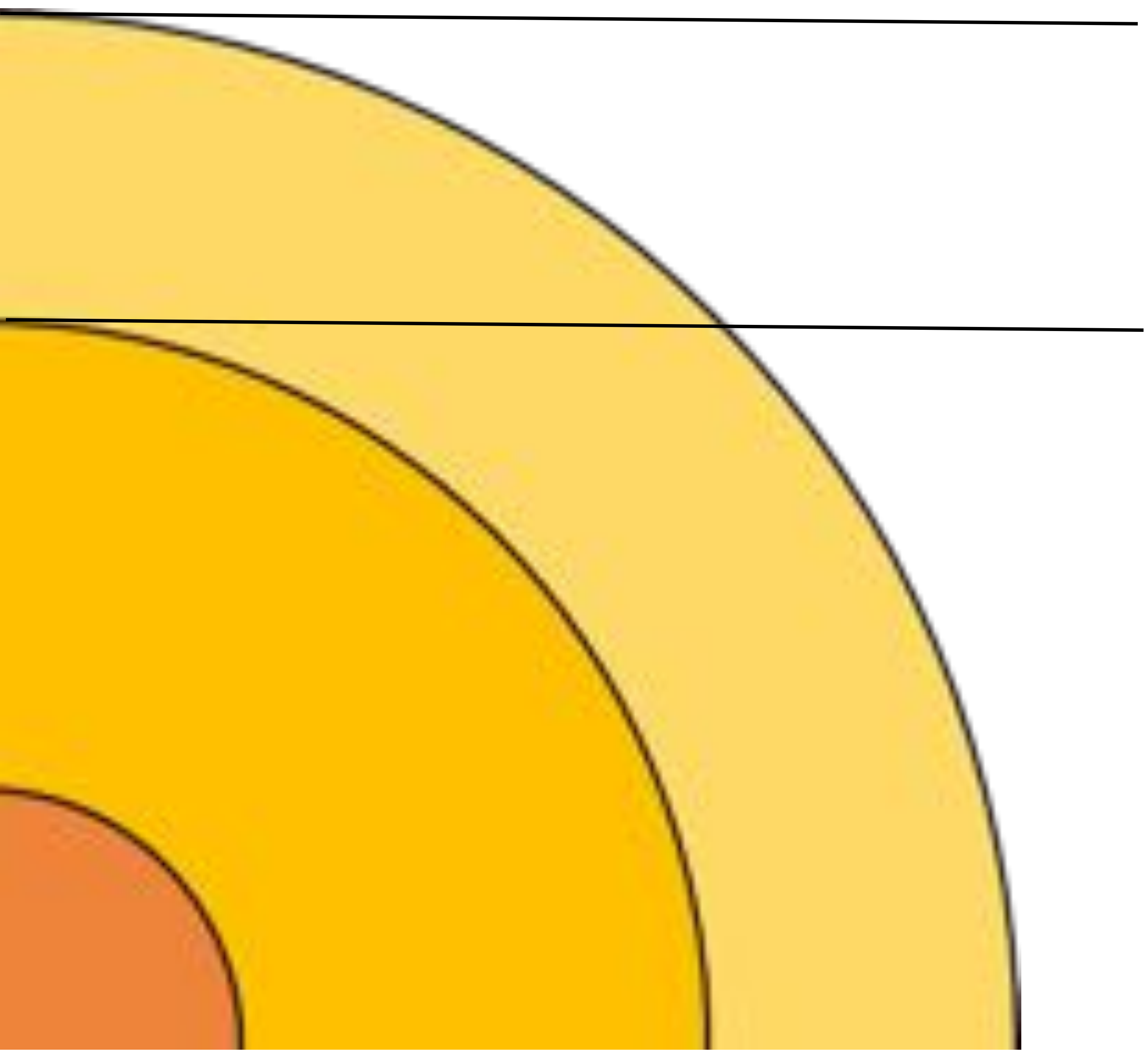
Convection simulations: heating from below, cooling from above

Numerical Models: The Challenge

$$\mathrm{Pm} = \frac{\nu}{\eta}$$

	Earth	Jupiter	Simulations
Ra	10^{31}	10^{37}	10^{6}-10^{7}
Ek	3×10^{-15}	10^{-9}	10^{-6} - 10^{-7}
Rm	300-1000	400-3×10^{4}	50-3000
Pm	5-6×10^{-7}	6×10^{-7}	0.1-0.01

The Sun is Even Worse…



Convection Zone Bulk

Temperature: 14,400K
Density: 2×10^{-6} g cm^{-3}

Temperature: 2.3 million K
Density: 0.2 g cm^{-3}

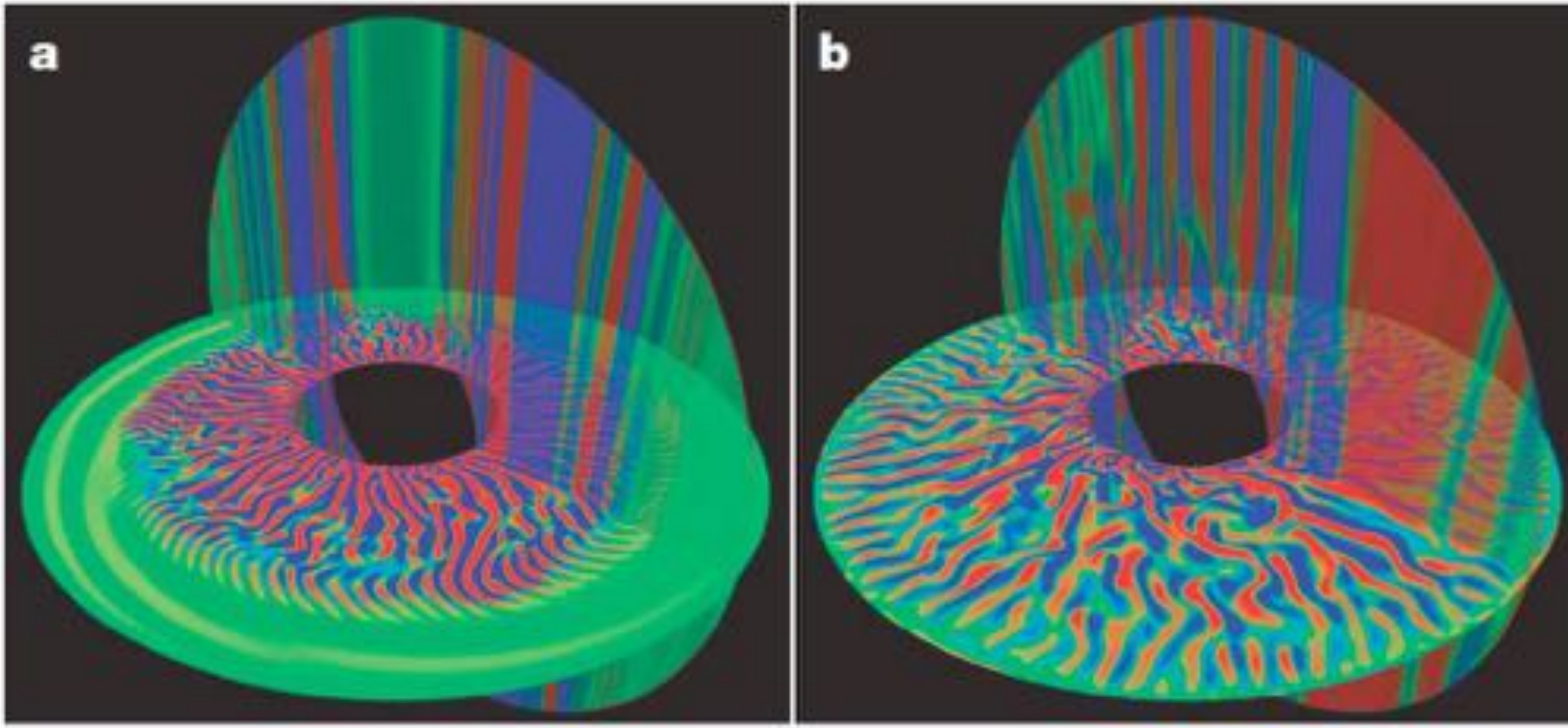
- 11 density scaleheights
- 17 pressure scaleheights
- Reynolds Number $\approx 10^{12} - 10^{14}$
- Rayleigh Number $\approx 10^{22} - 10^{24}$
- Magnetic Prandtl Number ≈ 0.01
- Prandtl Number $\approx 10^{-7}$
- Ekman Number $\approx 10^{-15}$

Nevertheless…

Axial alignment persists even in turbulent parameter regimes

Kageyama et al (2008)

Axial vorticity $\omega \cdot \Omega$



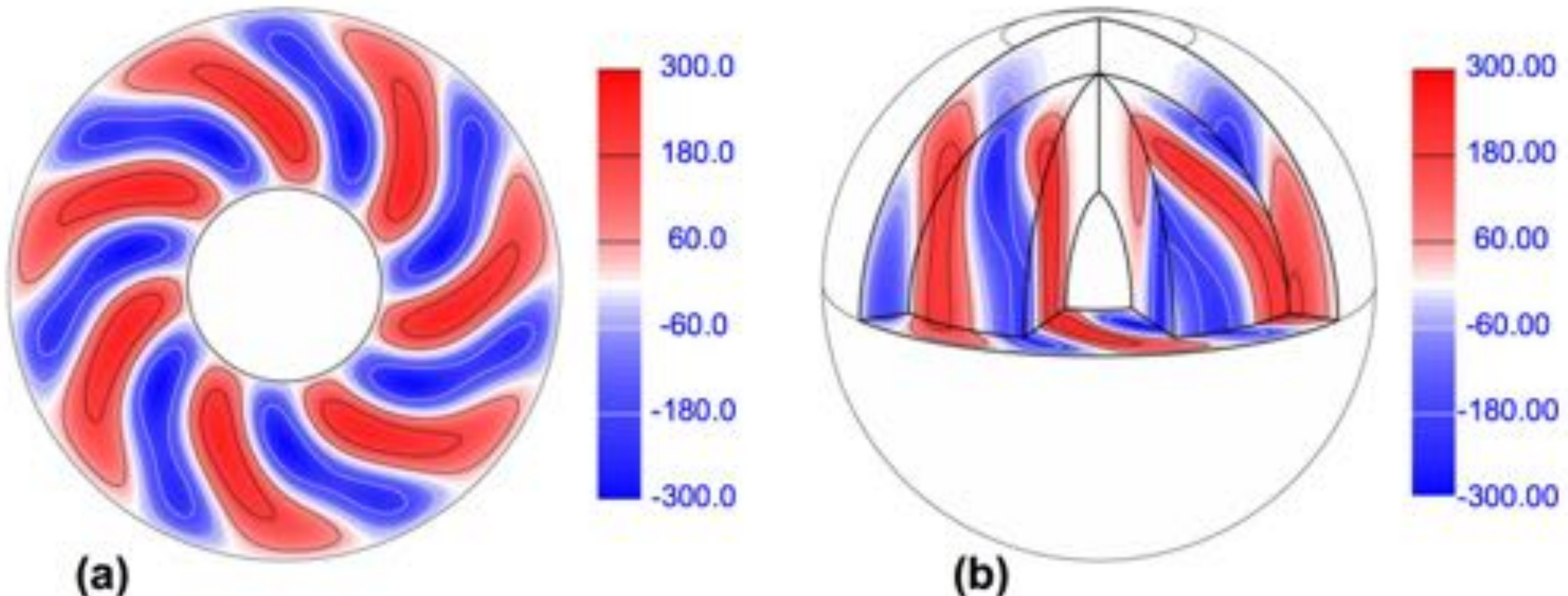
$\mathrm{Ek} = 2.3 \times 10^{-7}$ $\mathrm{Ek} = 2.6 \times 10^{-6}$

Busse columns give way to vortex sheets but the flow is still approximately 2D

$$\mathrm{Ek} = \frac{\nu}{2\Omega R^2}$$

…and in MHD

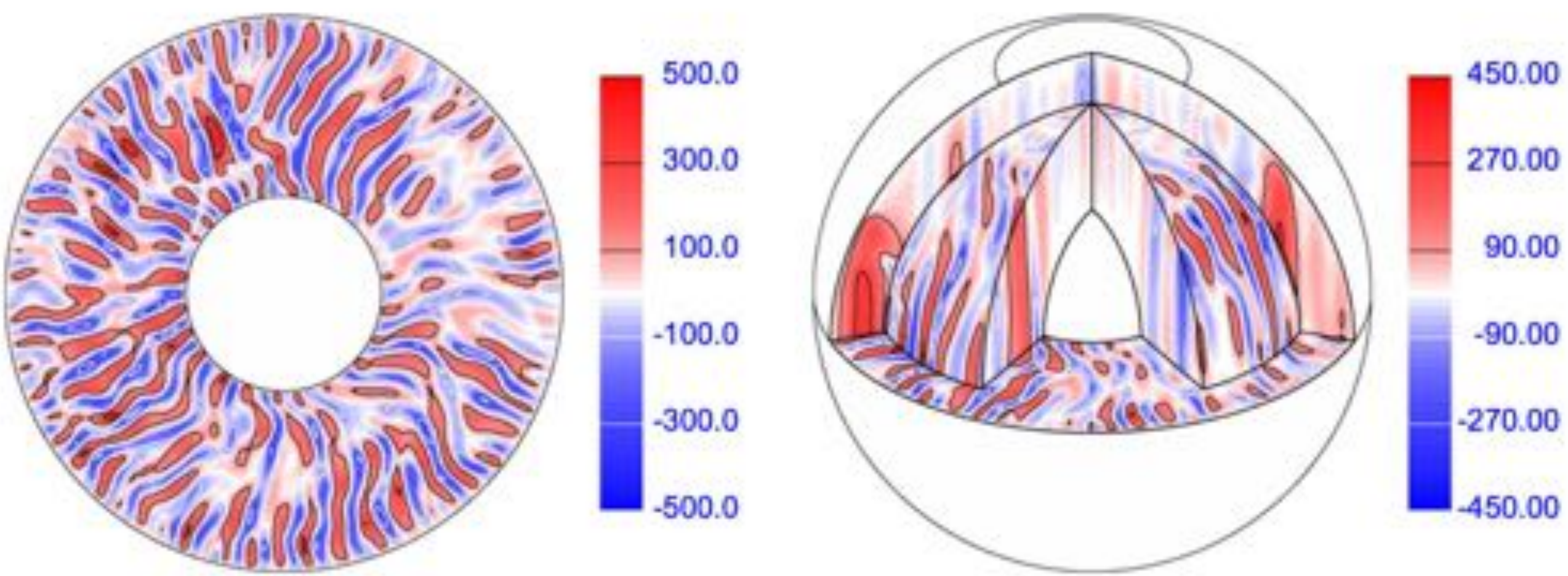
$$\mathrm{Ra} = \frac{GMD\Delta S}{\nu\kappa C_P} = \frac{\text{buoyancy driving}}{\text{dissipation}}$$



Ra = 8 ×10^5

Jones et al (2011)

Vr



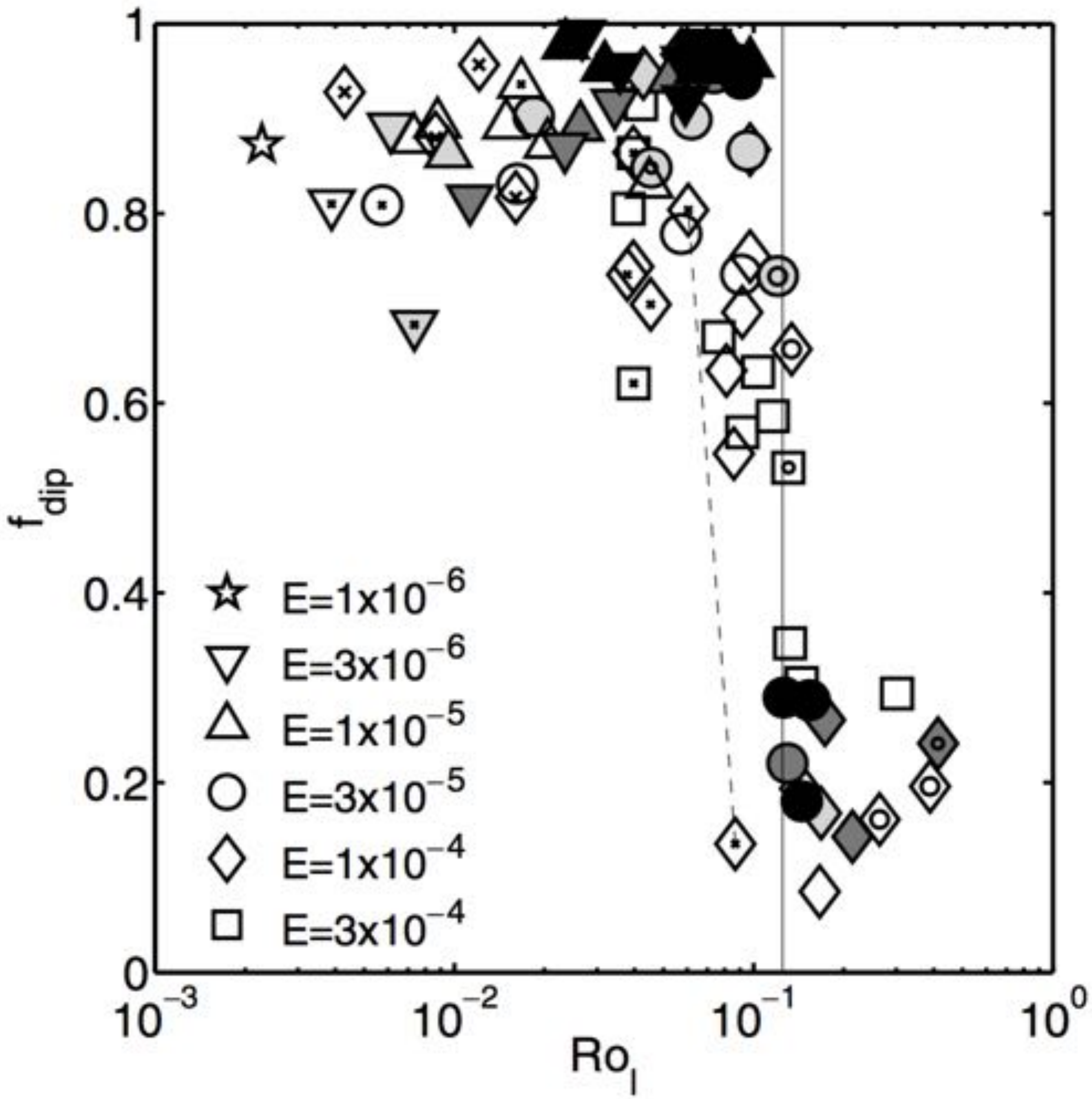
Ra = 2.5×10^7

General trends

Complexity of magnetic field depends mainly on the rotational influence

Rapid rotators tend to be more dipolar

Christensen & Aubert (2006)



Dynamical Balances

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\left(\rho \mathbf{v} \cdot \nabla\right) \mathbf{v} - 2\rho \left(\boldsymbol{\Omega} \times \mathbf{v}\right) - \nabla P + \rho \mathbf{g} + c^{-1} \mathbf{J} \times \mathbf{B} - \nabla \cdot \boldsymbol{\mathcal{D}}$$

Result: Flows evolve quasi-statically in so-called

Magnetostrophic (MAC) Balance

$$c^{-1} \mathbf{J} \times \mathbf{B} \approx 2\rho \left(\boldsymbol{\Omega} \times \mathbf{v}\right) + \nabla P - \rho \mathbf{g}$$

Question

$$c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\mathbf{\Omega}\times\mathbf{v}\right) + \nabla P - \rho\mathbf{g}$$

Assuming MAC balance, compute the ratio of ME/KE
How does it scale with Ro?

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J}$$

$$\mathrm{ME} = \frac{B^2}{8\pi}$$

$$\mathrm{Ro} = \frac{U}{2\Omega D}$$

$$\mathrm{KE} = \frac{1}{2}\rho v^2$$

Question

$$c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\mathbf{\Omega}\times\mathbf{v}\right) + \mathbf{\nabla}P - \rho\mathbf{g}$$

Assuming MAC balance, compute the ratio of ME/KE
How does it scale with Ro?

$$\frac{ME}{KE} \sim \mathrm{Ro}^{-1}$$

>>1 *if* Ro << 1*!*

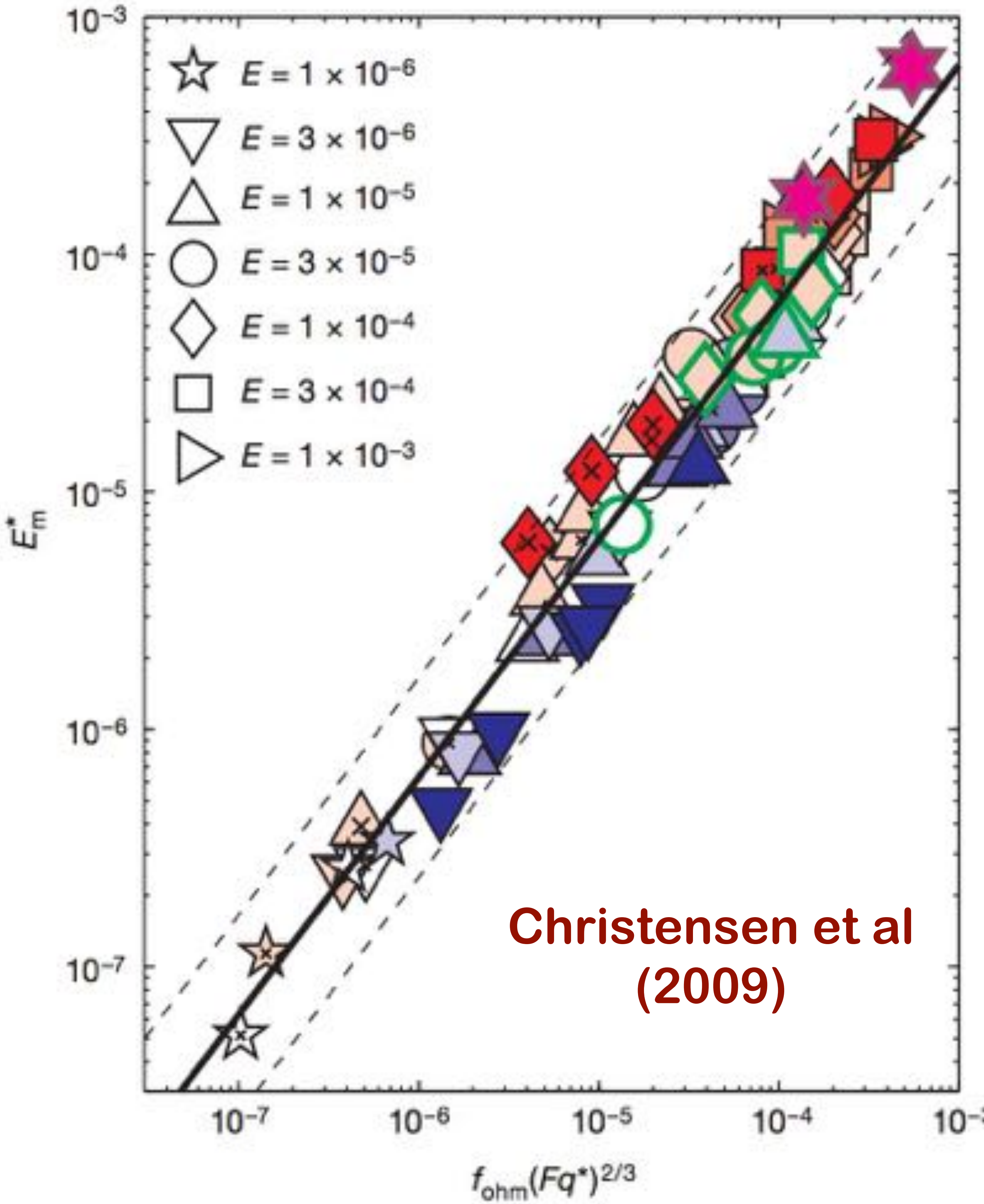
But how do KE and Ro (and thus, ME) depend on observable* global parameters like Ω and F_c?

**in principle*

General trends

Field strength scales with the heat flux through the shell (independent of Ω!)

Rapid rotators seem to operate at maximum efficiency, tapping all the energy they can



Christensen et al (2009)

Numerical Models: The Hope

Realistic simulations might be possible if you can achieve the right dynamical balances (e.g. MAC balance)

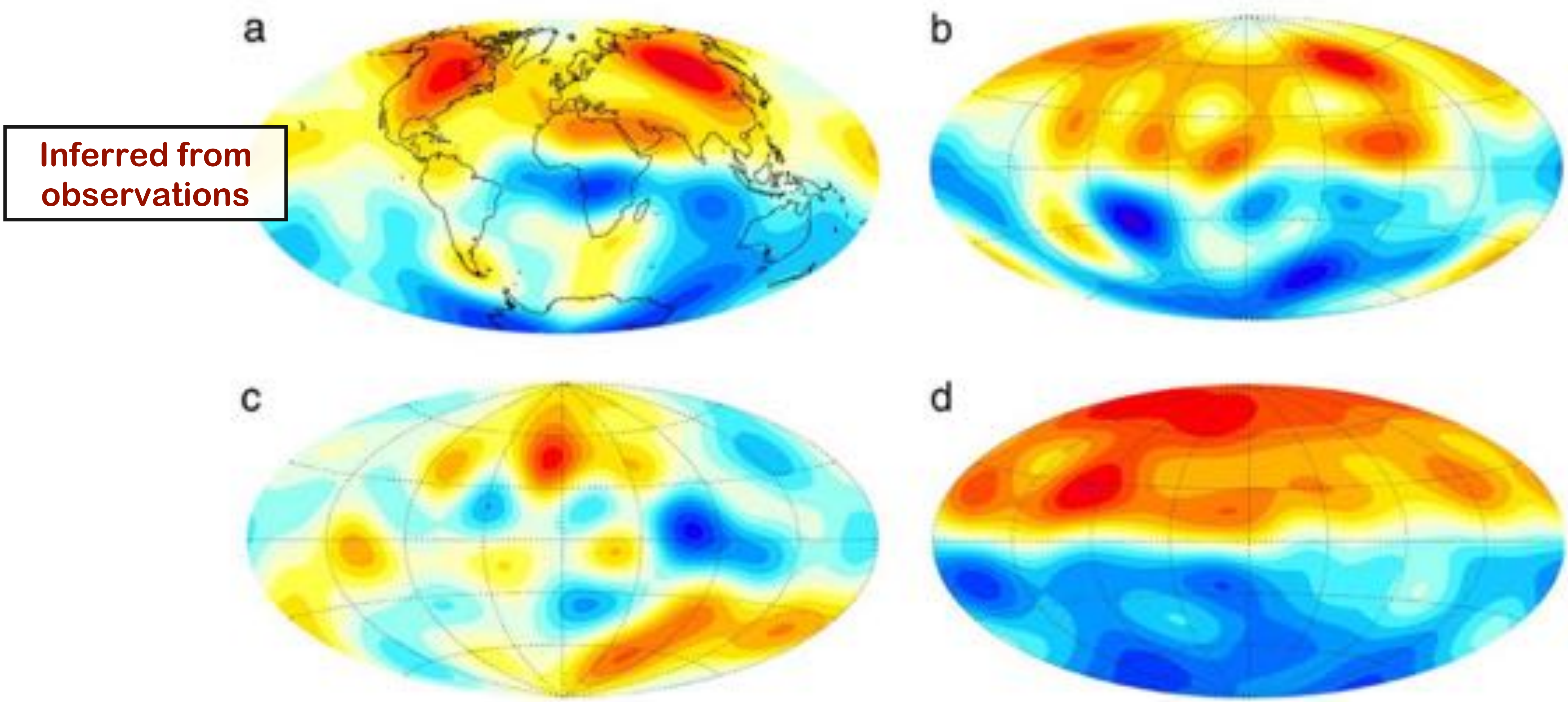
The most important parameters to get right

(or as right as possible)

- **Ro**
  - **Appropriate rotational influence on the convection**
- **Rm**
  - **Reasonable estimate of the ohmic dissipation**
- **Ek**
  - **At least get it small enough that viscosity isn't part of the force balance**

Example: The Geodynamo

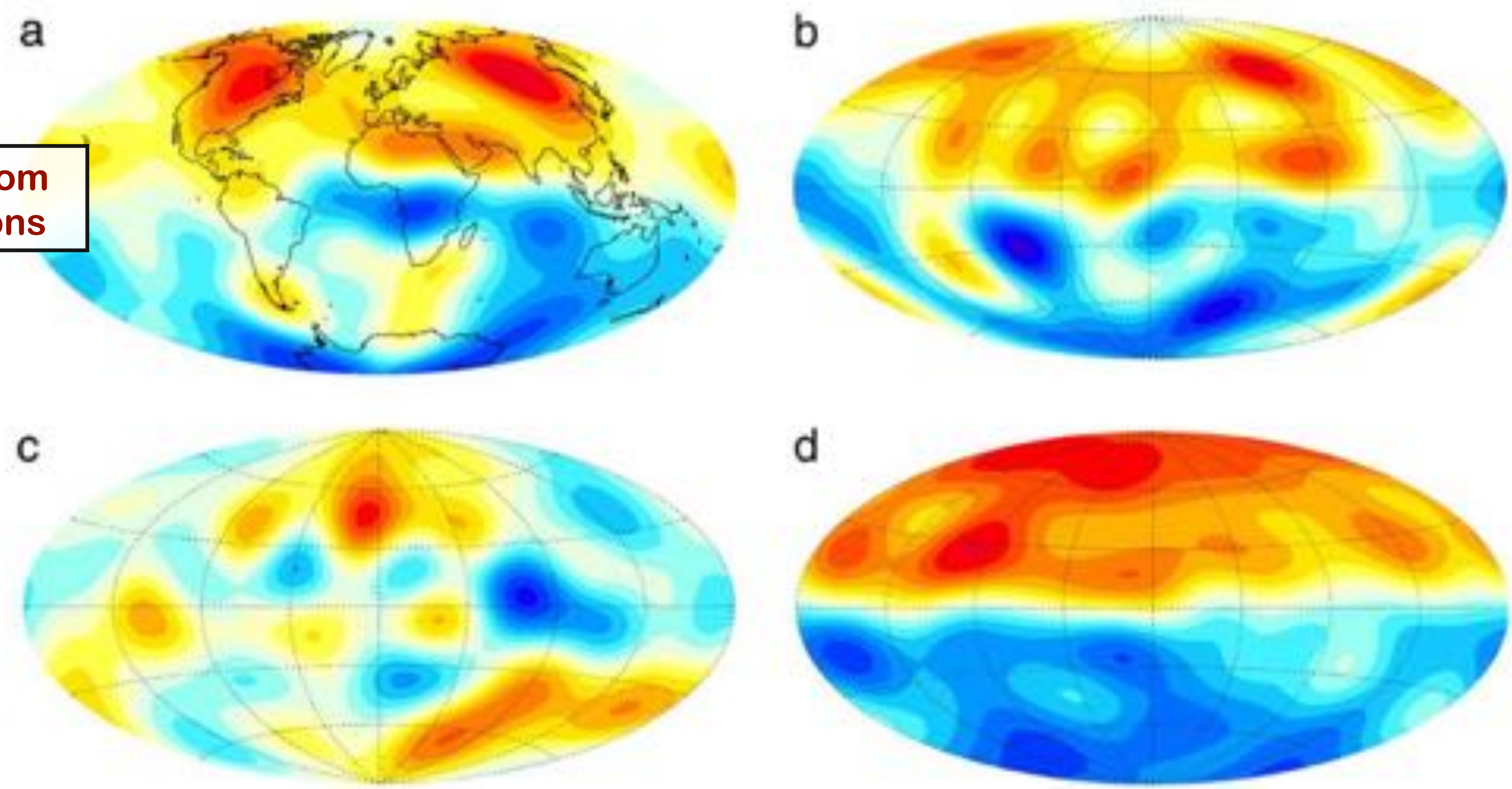
Points of comparison: Field strength, morphology (spectrum, symmetry, etc), Reversal timescale



Christensen et al (2010)

Best matches are those with Ek < 10^{-4} and Rm "large enough"

Example: The Geodynamo



Inferred from observations

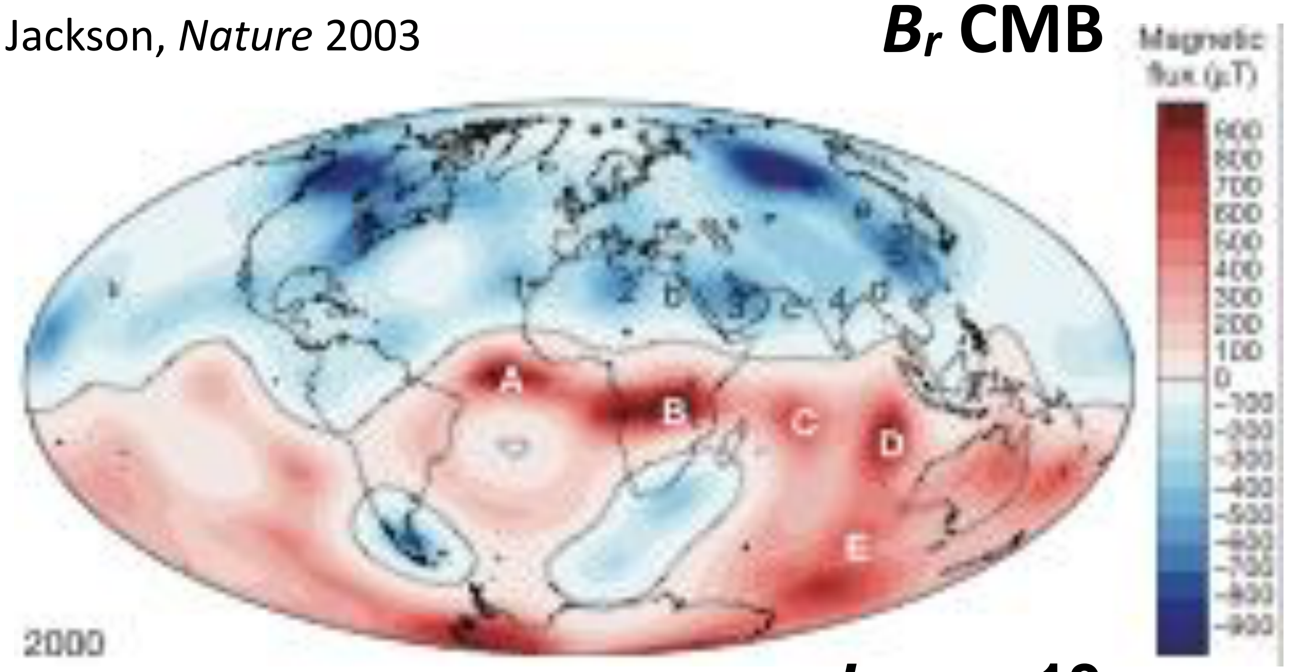
But be careful! They could be right for the wrong reasons!
For example, both c and d have a higher Ra and lower Ek than b they should be more realistic, right?

Observations

Models

Jackson, *Nature* 2003

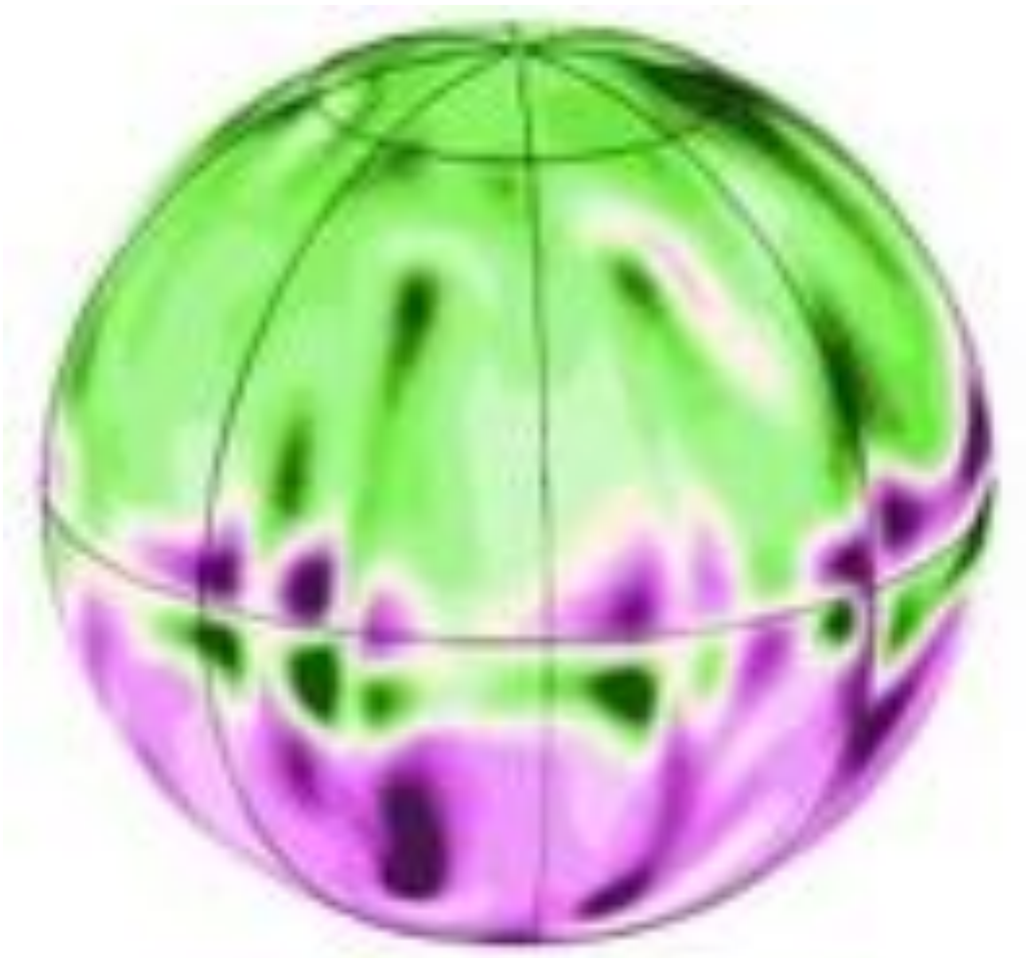
B_r CMB



L_{max} ~ 13

Soderlund et al. *EPSL* 2012

B_r CMB



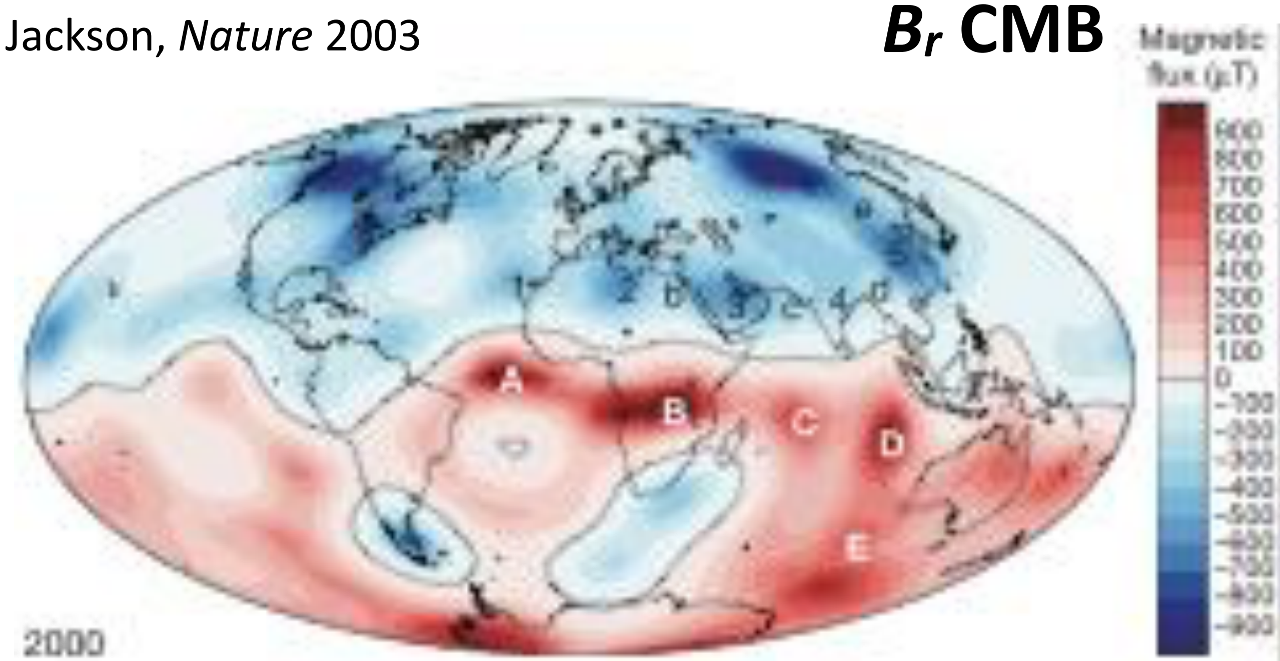
On the surface, things look pretty good…

Observations

Models

Jackson, *Nature* 2003

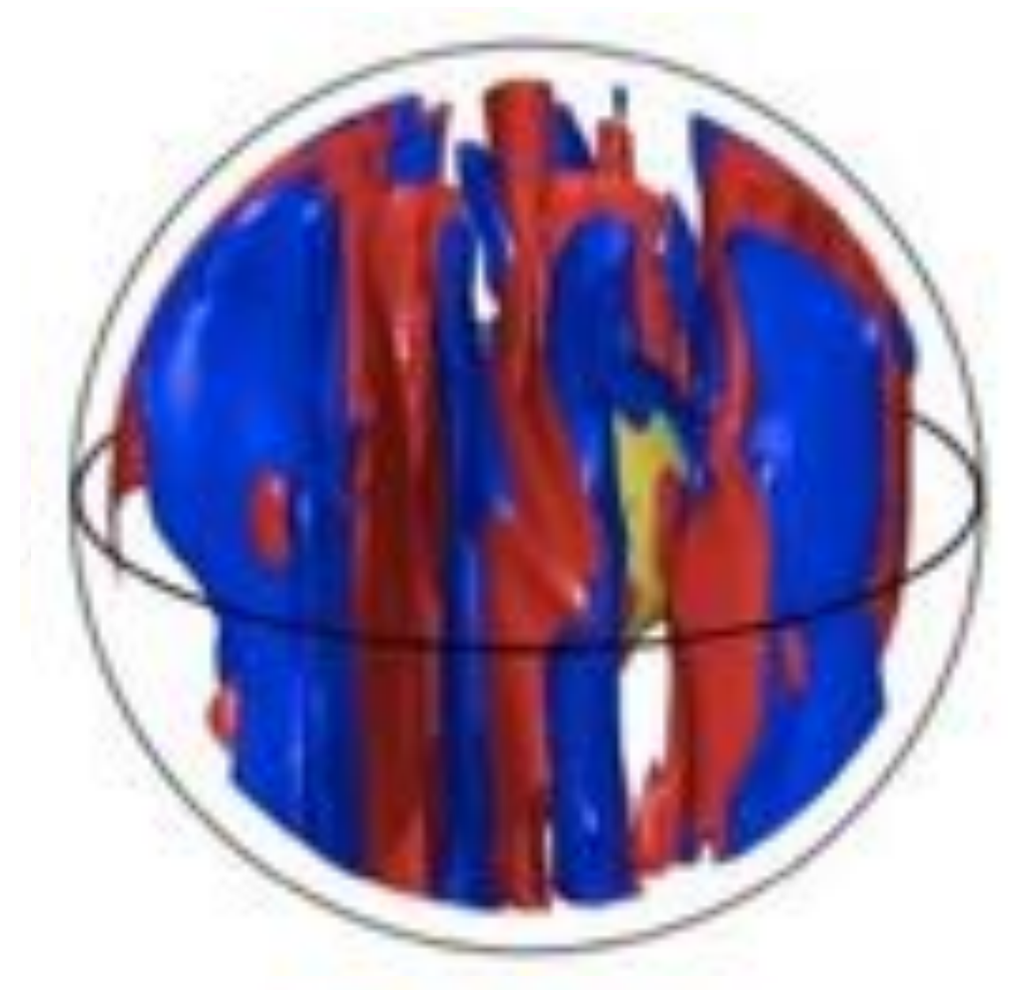
B_r CMB



L_{max} ~ 13

Soderlund et al. *EPSL* 2012

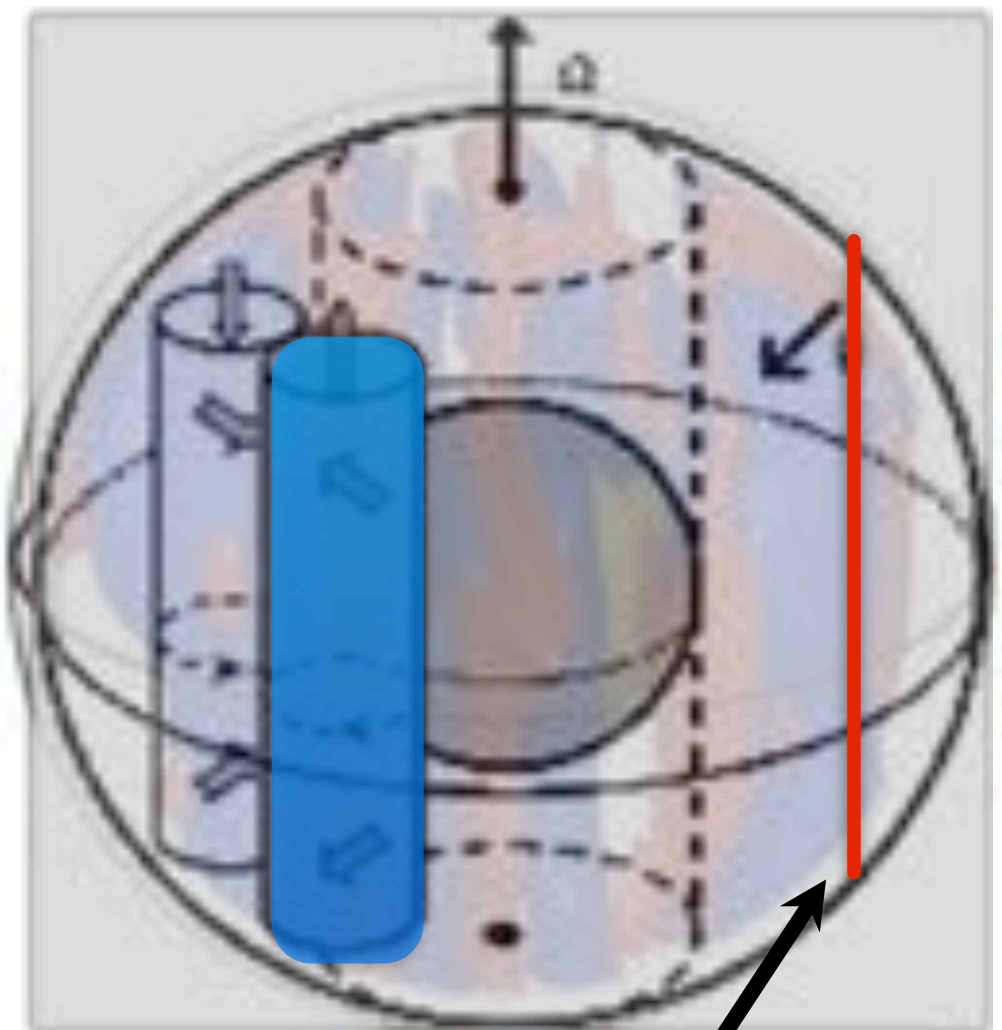
***z*-vorticity**



Beneath the surface …

… probably unphysical

Rotating Convection Columns:
column size set by Ekman number E



10^3
too wide

$$E = \frac{\tau_{rotation}}{\tau_{viscous}} = \frac{\nu}{\Omega L^2}$$

Models:

E ~ 1e-4; l_c ~ 0.1

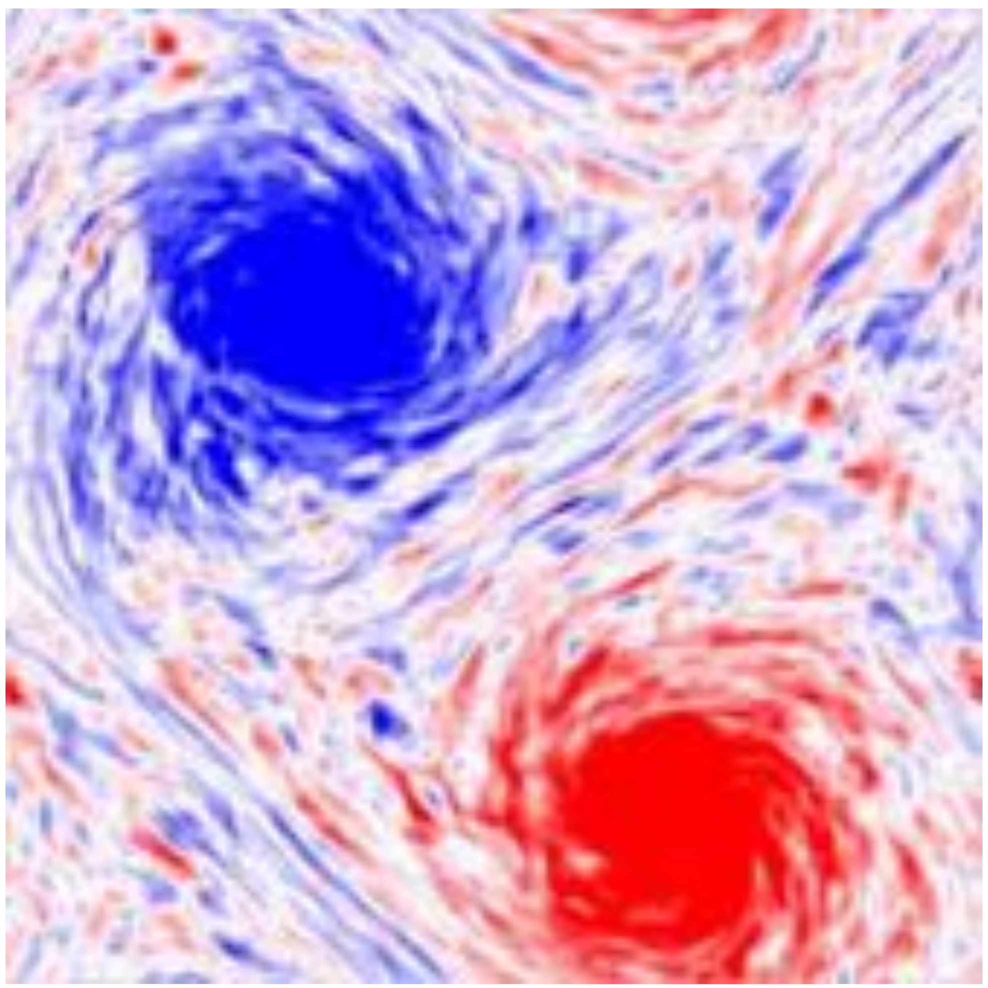
Earth's Core:

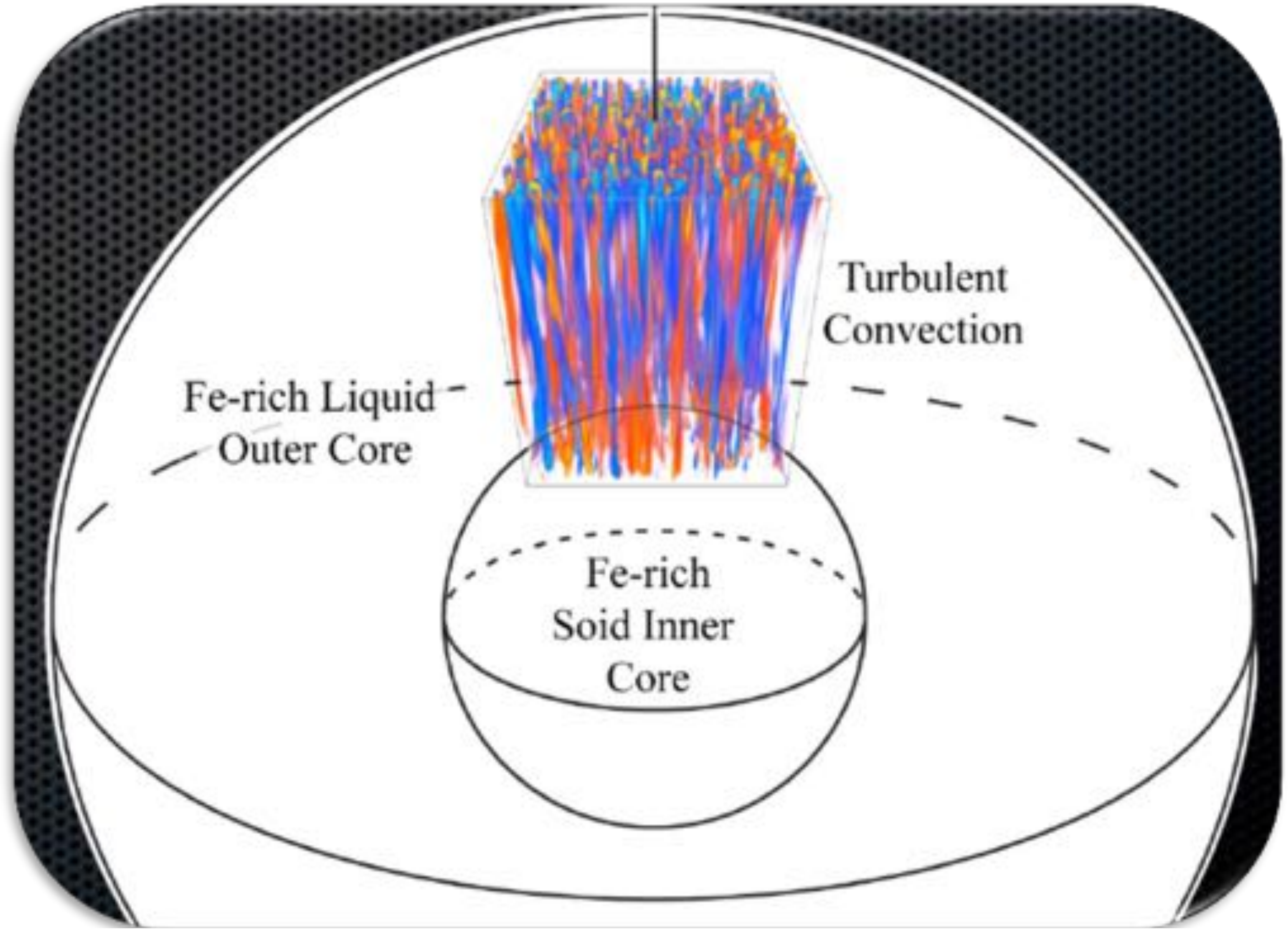
E ~ 1e-15; l_c ~ 1e-5

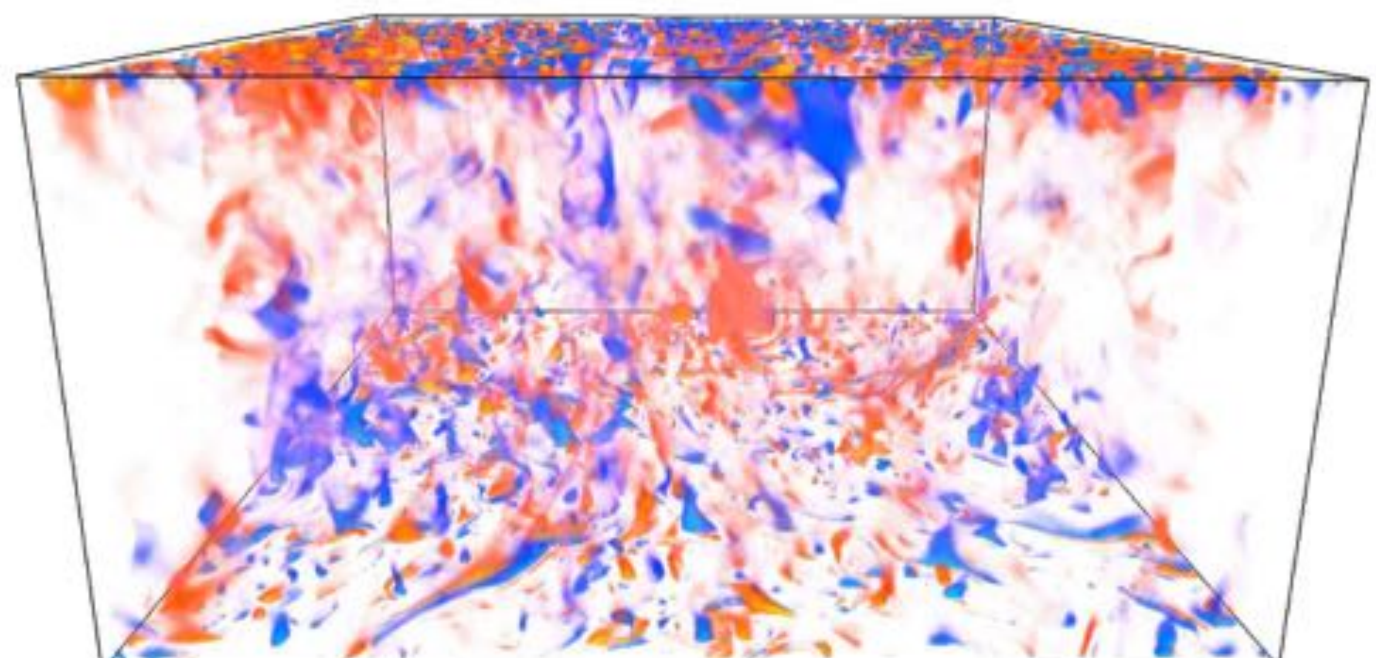
(i.e., 10^4 x smaller than scale of flux patches)

Rapidly Rotating MHD:

We observe large scales …
… but we know the small scale matter (a lot)





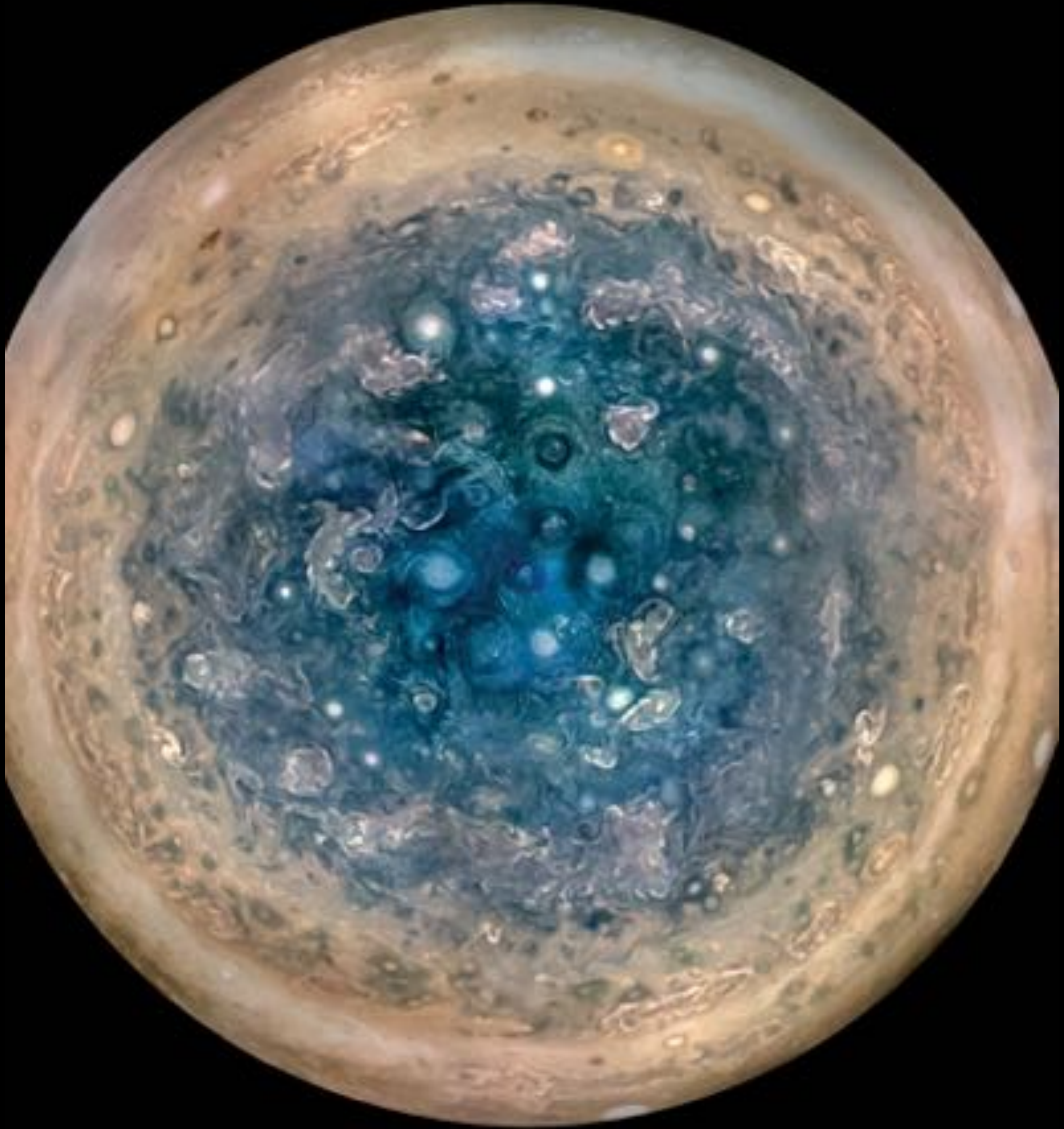


Rubio, Julien, Weiss, Knobloch PRL 2014

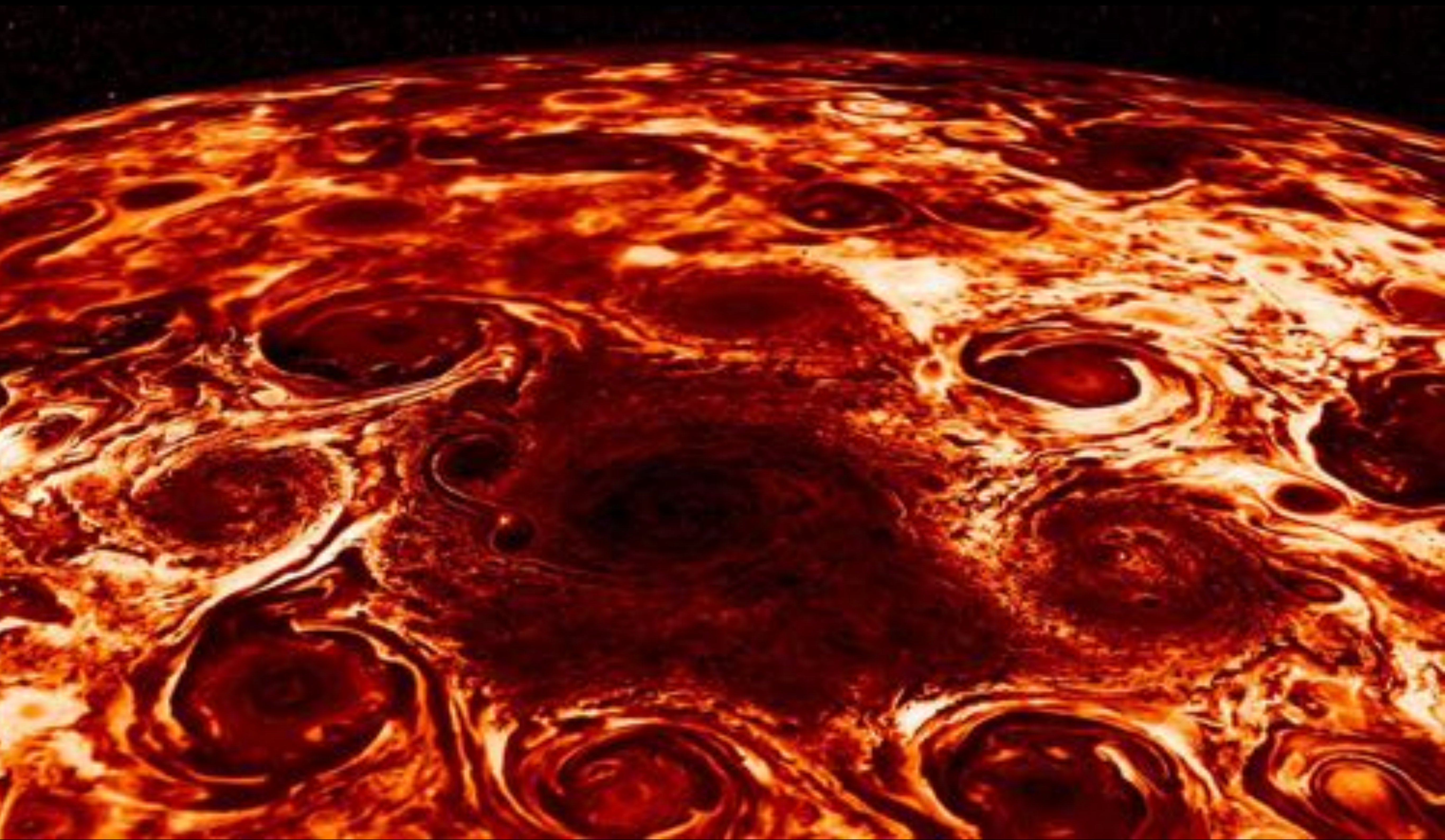
Numerical Models: Summary

- **Lessons Learned**
  - ***Rapid Rotation has a profound influence on the dynamics***
  - ***Success attributed to correct dynamical balances and (when possible) realistic Rm***
- **Future challenges**
  - ***What happens at really low Ek (tiny ν)?***
  - ***Peculiarities of particular planets (Saturn, Mercury, Uranus, Neptune…)***
    - **Boundary conditions (adjacent layers)**
    - **Rapid variations of η**
    - **Energy sources**
    - **Compositional convection**
  - ***Moving to more realistic parameters doesn't always improve the fidelity of the model***
  - ***Exoplanets!***

Juno!



Juno!



Heimpel et al. 2018
(in-prep)